THE UNIVERSITY OF ALBERTA

AN ECONOMIC ANALYSIS OF RATIONS FOR LACTATING DAIRY COWS

by

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A THESIS

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The undersigned certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "An Economic Analysis of Rations for Lactating Dairy Cows," submitted by Kenneth J. Nicol in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

The data for the analysis were obtained from an experiment conducted with the Department of Animal Science at the University of Alberta. The experiment was set up by selecting nine cows from each of three productive groups expected high producers, expected low producers, and first calf heifers. One cow from each of these groups was then placed in one of nine groups formed by supplementing three levels of roughage feeding to three levels of energy based on the National Research Council recommendations. The data collected allowed for the calculation of weekly fat-corrected and solids-corrected milk production and gave the kilograms of hay and grain consumed.

The initial phase of the analysis consisted of comparing the response of the cows within the three major divisions of the experiment on the basis of persistency, level of production, and body weight changes. The 100 and 120 percent energy levels were almost equal in persistency, and the 90 percent energy level group had a shorter lactation length. The lower levels of hay consumption were associated with a longer length of lactation and the higher levels of hay consumption with a shorter lactation length. From the three productive groups, the group designated as heifers had the longest lactation, then the group of expected high producers, and finally the low producers with the shortest lactation. The weight changes during the lactation were 38 (83.6), - 29 (-63.8), and -48 (-105.6) kilograms (pounds) for the 120, 100, and 90 percent energy levels, respectively. Total production of the three energy groups were 5,356 (11,773), 5,842 (12,855), and 4,282 (9420) kilograms (pounds) of 4 percent fat-corrected milk for the 120, 100, and 90 percent energy groups, respectively. The weight loss of the 100 percent group slightly affected total production, while the 90 percent energy group lost a larger amount of weight and had a lower level of production.

In determining the production functions, the data were grouped into consumption and production levels of four weeks duration commencing with week two and proceeding to and including week 37. The initial

analysis was done with a quadratic regression formula including hay and grain as the independent variables. This analysis produced maximizing equations for only six of the nine periods, and the level of significance associated with the coefficients was low. The proportion of the variation in the dependent variable (fat-corrected milk) that the equation explained was only in the region of 50 percent.

The isoquant analysis with the six equations gave varied results. Only the period weeks two to five had isoquants convex to the origin, and the remaining five periods produced isoquants that were linear or concave to the origin. The isoquants in each group indicated decreasing marginal rates of transformation; in other words, the distance between consecutive isoquants increased as the levels of the resources increased.

The analysis with solids-corrected milk as the dependent variable did not give coefficients that were significant to any higher level than in the fat-corrected milk analysis. The equations did not explain the variation in production of solids-corrected milk to any greater degree than when fat-corrected milk was the dependent variable. This analysis also failed to produce a maximizing equation for the 14 to 17 week period of lactation.

The price analysis was performed using the equation for the period weeks two to five. The analysis showed a direct relationship between the change in the optimum level of production and the price of milk. If the price of one of the resources increased, the level of use of that resource was reduced and the use of the other resource was increased, the level of output also being adjusted downward. The optimum level of production using the average prices of hay, grain, and milk were computed for each of the periods. The trend was for the level of production to be reduced with time and for a higher proportion of hay in the economically optimum ration.

The returns above feed cost were calculated for the optimum economic ration of the period, weeks two to five. The returns were then compared with the return of the maximum level of production and with two rations calculated from the National Research Council requirements and based on .75 kilograms and 1.5 kilograms of hay per 100 kilograms of body weight, respectively. The optimum economic ration had the highest return above feed costs of $66.20 followed by the 1.5 kilograms hay ration then the

.75 kilograms of hay ration and the maximum production level ration with returns of \$61.12, \$58.39, and \$52.90, respectively. Price changes of the milk and feed components were then compared for the optimum economic ration and the two National Research Council based rations. With the price changes the need for changes in the level of production and amounts of feed provided became evident. The optimum economic ration gave a return over feed costs that ranged from \$7.43 to \$11.80 above the .75 kilogram of hay ration and from \$6.89 to \$9.49 above the 1.5 kilograms of hay ration.

ACKNOWLEDGEMENTS

The writer wishes to thank Dr. C. M. Grieve and the Department of Animal Science for providing the data, Dr. H. C. Love for his guidance in the initial analysis, Dr. A.A. Warrack for his help in the final analysis and the preparation of this thesis, and Mr. C. A. Shier for his help in the computational procedures.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

Background and Objectives

The dairy industry in Western Canada is approaching a crisis period. With the introduction of substitutes for milk and milk products the relatively stable position enjoyed by the dairy industry in the past seems almost certain to be upset. One of the alternatives available to the milk industry is to produce and offer for sale high quality products at competitive prices. In order to keep the price competitive the raw materials must be made available to the processor at as low a price as possible and still provide the producer with a return for his labor and capital in line with their respective opportunity costs. Mechanization has led the way to the development of physical plants that reduce the fixed costs and labor requirements per unit of milk produced. What then can be done about the reduction of the variable costs represented by feed consumed by the cow? Are feeding standards designed to provide economical rations which maximize profit? With these questions in mind an experiment was conducted at the University of Alberta to determine the response of the dairy cow to varying feed combinations and to determine which combination of hay and grain are the most economical at various stages of lactation.

Organization of the Thesis

The response of the cows to their feed was analyzed using a quadratic regression equation. The lactation was divided into 4 week periods upon which the regression analysis was performed with fat-corrected milk as the dependent variable for the major analysis. The coefficients from the regression equations were then used to determine the marginal rates of substitution of hay for grain at various levels. The coefficients were also combined with representative prices to determine the optimum economic ration.

The return above feed costs was then compared for the optimum economic ration and two rations computed on the basis of the National Research Council requirements.[1] From this comparison the economic potential of the rations calculated from the National Research Council requirements was defined. The last section of the thesis deals with the results and the implications that the results provide as to the feasibility of the existing recommendations. The implications as to further work which could be undertaken as a follow up to this study are then outlined.

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1 National Academy of Sciences, National Research Council <u>Nutrient Requirements of Dairy Cattle</u>, Publication 1349 (Washington D.C. : NAS, NRC, 1966), pp.2-6

CHAPTER II

ECONOMIC PRINCIPLES

Graphical Analysis

The production of a good M, from a set of resources X_1, X_2, X_3, ..., X_n can be represented by the general long run production function

(1) $M = f(X_1, X_2, X_3, \ldots, X_n)$.

If M represents the milk produced in a specified period of time, X_1 the hay consumed in the period, X_2 the grain consumed in the period, X_3 the labor used during the period, X_4 the cows age, X_5 the period of the lactation and $X_6 \ldots X_n$ other variable factors of production, then equation 1 could represent a general production function for milk. In this project most of the analysis was carried out with only two independent variables hay and grain with all others stabilized as well as possible. In this manner the general short-run production function took on the form

(2) $M = f(H, G | X_3, X_4, \ldots X_n)$

where H represented hay intake (X_1) and G represented grain intake (X_2) over the period of the study. In this function only those variables represented to the left of the vertical bar are considered as independent variables. The effects of the variables listed to the right of the vertical were residual, and their influence on the dependent variable was not considered in the analysis.

Equation 2 can be represented on a three dimensional figure. The base of the figure consists of the quantity of hay on one axis and the quantity of grain on the other axis. The third dimension depth, represents milk production forthcoming from all possible combinations of grain and hay offered to a lactating cow - a production surface. At each point on the feed intake surface a level of production was specified representing the quantity of milk that a cow would have produced while consuming the amount of hay and grain offered. All these points constitute a milk production surface for the intake combinations of hay and grain. On the production surface the various contours illustrate the variation in the substitution rate of hay for

grain, and the isoclines over it determine the rate of transformation of a hay-grain combination into milk. Using these two procedures, the lowest feed ratio can be determined for any given level of milk production. The isoclines can also be used to determine the level of feeding to allow for maximum returns over feed costs per cow.

A production function such as equation 2 can take several algebraic forms; it may contain linear terms. Some of the functions that will be considered are the quadratic, square root, and logarithmic forms. A linear function gives a production surface Figure 1, which has a constant marginal rate of transformation of hay or grain into milk[1] In the linear case the isoquants (I, II, III) are straight lines running across the surface and the isoclines, (OA, OB, OC, OD, OE) take on a fan shape up through the isoquants, Figure 2.

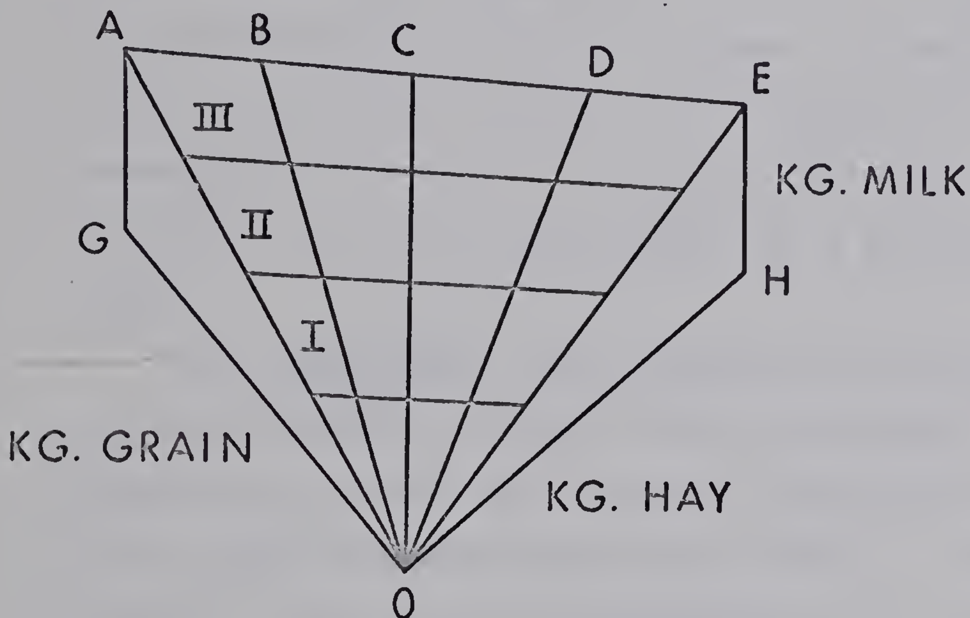

Figure I

PRODUCTION SURFACE FOR A LINEAR PRODUCTION FUNCTION

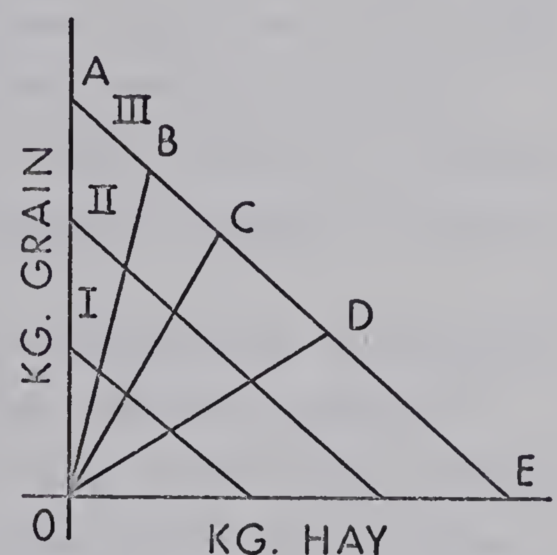

Figure 2

ISOQUANT AND ISOCLINE MAP FOR A LINEAR PRODUCTION FUNCTION

Using this analysis the optimum ration would be at one end of the isoquants depending on the ratio of the price of hay to the price of grain. OA and OE would be the only relevant isoclines in this

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1 E.O. Heady, J.A. Schnittker, N.L. Jacobson and S. Bloom., Milk Production Functions, Hay/Grain Substitution Rates and Economic Optimum in Dairy Cow Rations Bulletin 444. Ames: Iowa State Agricultural Experiment Station, 1956, p.897

analysis, and the position chosen for production on OA or OE would depend on the ratio of the price of the relevant feed component to the price of milk. If the ratio of the price of the feed component to the price of milk is greater than the transformation ratio, then production will be at level zero. If the ratio of the price of the feed component to the price of milk is less than the transformation ratio the cow should be fed to her capacity of the feed component, level H for hay and level G for grain, and depending on whether the component is hay or grain, the relevant point of production will be at A or E respectively. Using this type of analysis only three points on the production surface have any significance; they are point 0, point E, and point A.

An analysis on the basis of a linear production function may hold true over a small range of the true production surface, but to assume that it will hold over the entire range of possible feed inputs and milk output indicated by the function is not possible, as some limit must occur to consumption as well as to milk production. For these reasons the linear function will not be considered as a possible method of analysis in this study.

The logarithmic style function can also be eliminated by an analysis of the production surface that it produces. This function produces a production surface that becomes asymptotic to a limit rather than allowing for a single maximum production level [1]. The logarithmic function also does not allow for the determination of a single ration that will produce the maximum amount of milk but rather provides a range of rations that produce equally well as the limit is approached. The isoclines determined from the logarithmic production function are linear, and they fan out over the range of rations rather than converge to a point of maximum production. This type of function limits the substitution of the factors as the isoquants become asymptotic to the axis, and one of the factors cannot be completely substituted for by the other factor as one possibility in the range of production possibilities.

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1 E.O. Heady and J.L. Dillon , Agricultural Production Functions (Ames: Iowa State University Press, 1961), P.85

An isoquant and isocline map for the logarithmic function is diagrammed in Figure 3 with OA, OB, OC, and OD representing isoclines and I, II, III representing the isoquants. This map is similar in principle to Figure 2, except that with the logarithmic function the isoquants are curved and do not approach the axis whereas in Figure 2 the isoquants were linear and proceeded to the axis.

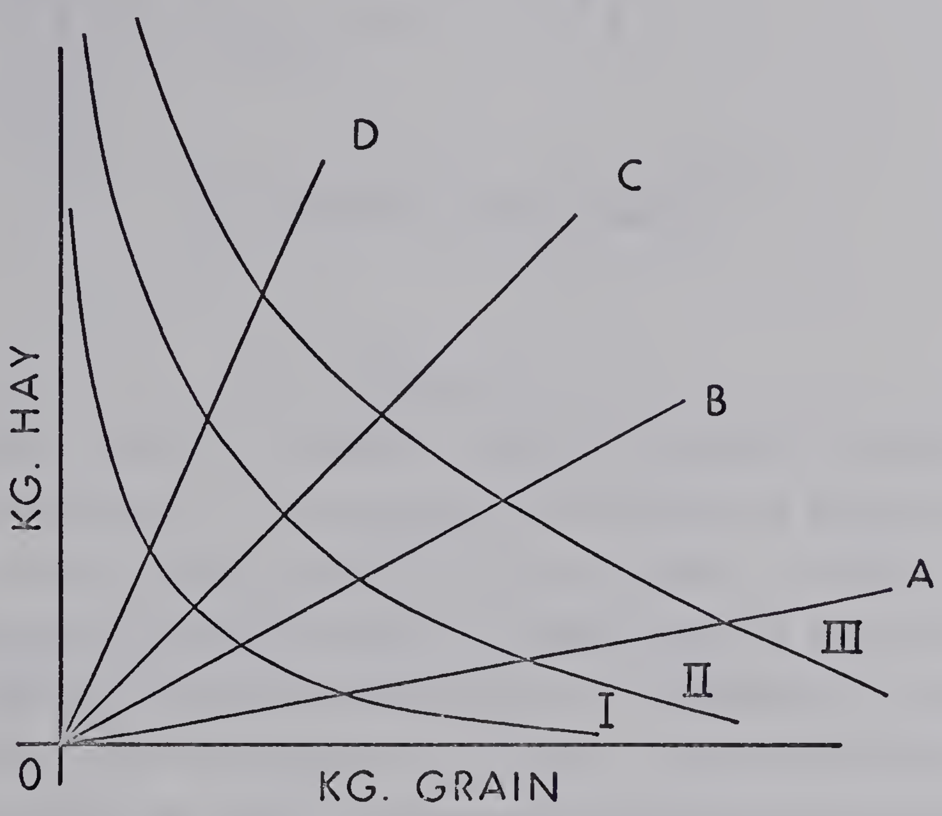

Figure 3
ISOQUANT AND ISOCLINE MAP FOR A LOGARITHMIC PRODUCTION FUNCTION

The quadratic and square root production functions provide feed input/milk output lines (Figure 4) where R_1, R_2, R_3, and R_4 each represent the production possibilities curve of a correspondingly less productive ration. The quadratic and square root function both have non-linear isoquants that allow for the complete substitution of one of the ration components for the other as one alternative in the range of production possibilities.[1]

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1 Ibid., p 78.

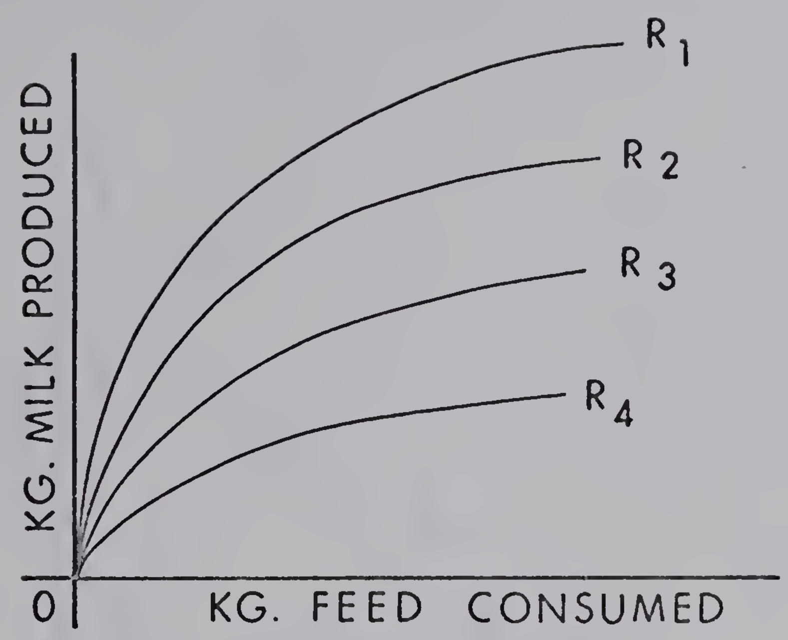

Figure 4

TOTAL PHYSICAL PRODUCT CURVES OF VARYING RATIONS

The curvature of the isoquant indicates a diminishing rate of substitution of one feed component for the other. Figure 5 illustrates the curved isoquants and isoclines. From Figure 5 note that the ratios R_1, R_2, R_3, and R_4 increase production at a diminishing rate as the distances between the isoquants I, II, III, and IV increase. Each isoquant represents an equal incremental increase in production, yet each must move farther out on the feed surface than the last to achieve the increase; that is $OR < RS < ST < TU$, while the quantities represented by I, II, III, and IV are incremental. This system of analysis allows for a maximum level of production produced by only one combination of feed components and also exhibits diminishing returns to feed, representing the approximate response of a cow to its feed.

ALGEBRAIC ANALYSIS

A general quadratic equation that represents the previous analysis is

(3) $$M = a_o + a_1H - a_2H^2 = a_3G - a_4G^2 - a_5HG$$ [1]

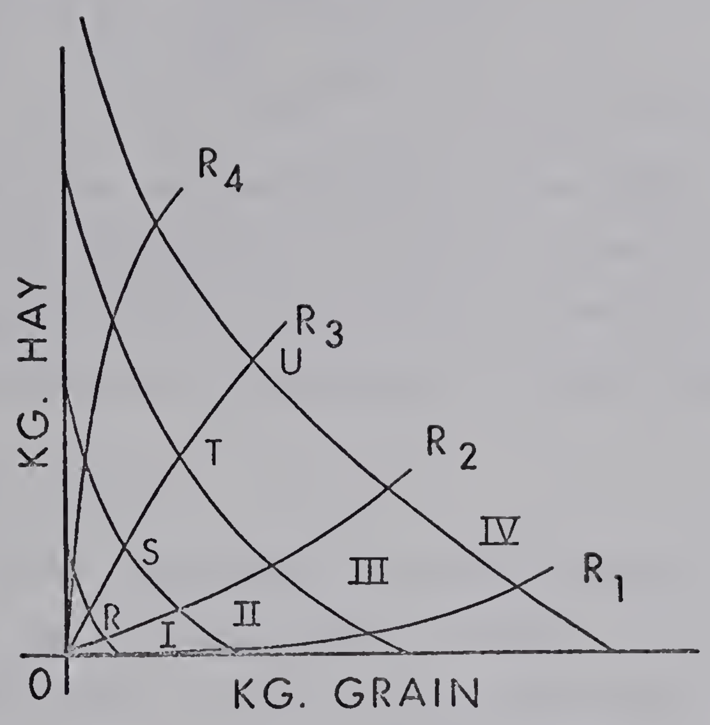

Figure 5

ISOQUANT AND ISOCLINE MAP

FOR A QUADRATIC PRODUCTION FUNCTION

where M = Kg of milk produced, H = Kg hay consumed, and G = Kg of grain consumed all in the same given time period. The subscripted 'a' values represent the coefficients to be estimated by regression analysis of the given set of data for the specified period. The function can be used to derive the production surface by substituting in set values for H and G and computing M. When a sufficient number of these calculations are made, they can be plotted on a three dimensional figure, and the production surface determined with M = f(H,G), (Figure 6). Similarly by slicing through the production surface at

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This form was used for milk production function studies by E. O. Heady, J.P. Madden, N.L. Jacobson, and A.E. Freeman. Milk Production Functions in Relation to Feed Inputs, Cow Characteristics and Environmental Conditions. Bulletin 529 Ames: Iowa State Agricultural Experiment Station 1964 and also by L.M. Hoover, P.L. Kelley, G.M. Ward, A.M. Fayerhern, and R. Chadda. "Economic Relationships of Hay and Concentrate Consumption to Milk Production" Journal of Farm Economics XLIX (1967), 65.

any given level of H, H*, the single variable production function can be determined for $M = f(G|H^*)$, (Figure 7). Proceeding from equation 3 then $M = f(G|H^*)$ would be represented by

(4) $M = a_6 + a_7G - a_4G^2$

where $a_6 = a_o + a_1H^* - a_2(H^*)^2$ and $a_7 = a_3 \pm a_5H^*$. From this single variable equation one can determine the rate at which grain is transformed into milk by determining the slope of the function of Figure 7. The slope is represented by the first derivative of the algebraic equation (4) and is referred to as the marginal product equation. The marginal

(5) $\frac{\partial M}{\partial G} = a_7 - 2a_4G$

product of grain determined through the use of this equation is only relevant at the specified level of Hay, H*. In order to compensate for varying levels of hay feeding the partial derivative of milk with respect to grain (equation 6) is determined from equation (3). The

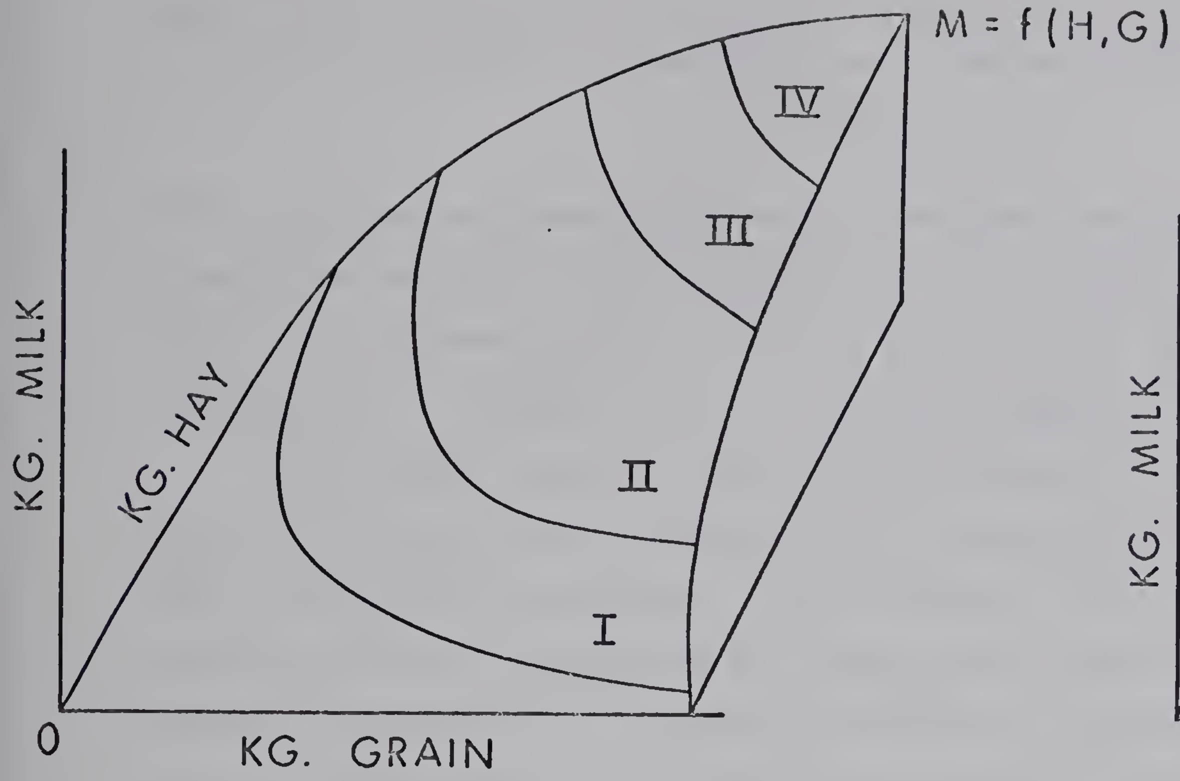
(6) $\frac{\partial M}{\partial G} = a_3 - 2a_4G \pm a_5H$

partial derivative gives the amount of change in milk production resulting from a change in grain consumption for any level of hay feeding used in the equation. This analysis holds also for the function $M = f(H|G)$ giving

(7) $\frac{\partial M}{\partial H} = a_1 - 2a_2H \pm a_5G$

Equations 6 and 7 determine the marginal productivities of grain and hay, respectively, when the other is held constant at a specified level. The equations also represent the slope of the total product curve denoted by $M = f(G|H)$ and $M = f(H|G)$, respectively. In order to determine the maximum physical product, the curve denoting total physical product must attain a horizontal position. The horizontal position denotes a zero marginal physical product (MPP), and thus the maximum physical product occurs when

(8) $MPP_G = \frac{\partial M}{\partial G} = a_3 - 2a_4G \pm a_5H = 0$



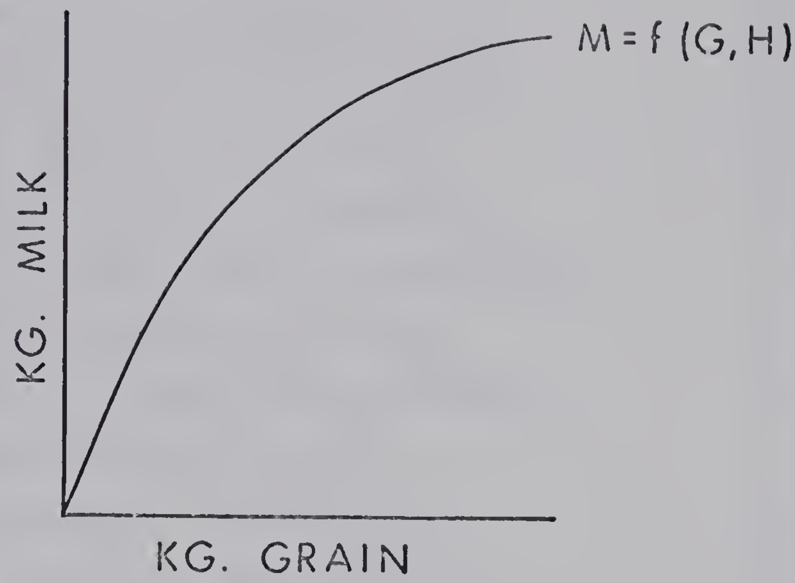


Figure 6
PRODUCTION SURFACE FOR M= f(H,G)

Figure 7
SINGLE VARIABLE PRODUCTION FUNCTION
$M = f(G|H)$

and

(9) $MPP_H = \frac{\partial M}{\partial H} = a_1 - 2a_2H + a_5G = 0.$

By taking the second derivative of equation 3, it is possible to derive the position at which the production function changes from an area of increasing marginal returns to an area of decreasing marginal returns. Setting the second derivative to zero and solving would give the position of the of the inflection point. Setting the two second derivatives,

(10) $\frac{\partial^2 M}{\partial G^2} = -2a_4$ and $\frac{\partial^2 M}{\partial H^2} = -2a_2$

determined, as equation 10, equal to zero produces undefined inflection points.

By rearranging the production function equation 3 in such a manner that the level of one resource is expressed as a function of the output and the level of the other resource the product contour or the isoquant

equation is determined to be

$$(11)\quad H = \frac{-(a_1 \pm a_5 G) + \{(a_1 \pm a_5 G)^2 - 4(-a_2)(a_o + a_3 G - a_4 G^2 - M)\}^{1/2}}{2(-a_2)}$$

This equation mathematically expresses the contours or isoquants on a production surface. The isoquants are determined by setting the milk output at a given level M and then solving for the hay value which corresponds to varying levels of grain substituted into the equation.

Once these contours have been derived, the next step is to determine the rate at which the resources, hay and grain, substitute for each other along these contours. This marginal rate of substitution between resources can be determined by taking the first derivative of the isoquant equation (11). Another method of determining the marginal rate of substitution is to take the negative inverse of the ratio of the marginal products of equation 3 and rewrite as

$$(12)\quad \frac{\frac{\partial M}{\partial H}}{\frac{\partial M}{\partial G}} = -\frac{a_1 - 2a_2 H \pm a_5 G}{a_3 - 2a_4 G \pm a_5 H} = \frac{\partial G}{\partial H}$$

In order to determine a set of points on a production surface with equal substitution ratios, the isocline equation is developed from the equation for the marginal rate of substitution. Initially set K equal to the desired substitution ratio and then equate K to the marginal rate of substitution without the negative sign. Transposing the equation and solving for the quantity of hay, the isocline equation is

$$(13)\quad H = \frac{a_1 - Ka_3 + (a_5 - 2a_4 K)G}{a_5 K + 2a_2}$$

From this equation the isoclines can be determined by setting K at the desired substitution ratio (K*) then with grain set at various levels solving for the corresponding levels of hay. This set of hay and grain coefficients then denotes the isocline corresponding to a substitution ratio equal to K*; by changing K* to K** and solving for the value of hay that now corresponds to the values of grain, another isocline can

be developed for the substitution ratio K**. Similarly any pathway can be determined at which output increases within the bounds of a given substitution ratio.

Limitations

The production surface under unlimited resource use has now been developed, and the substitution ratios determined in the form of the isoquants and isoclines. What alterations must now be made to adopt the theory of production to a limiting technical unit such as the dairy cow? There exists a maximum amount of feed that she can consume due to the limiting size of her rumen and the rate of movement of the food through the rest of the digestive tract. Also there exists some physiological minimum amount of hay which must be fed to maintain the proper functioning of the rumen. To incorporate these factors into production, some means of expression of maximum capacity must be specified. The maximum intake function will form a boundary on the production surface such that $C = f(H_F, G_F)$ where C = maximum capacity and H_F and G_F represent the filling capacity of hay and grain, respectively.[1] Similarly another boundary will be formed above the grain axis indicating some minimum physical quantity of hay which must be fed.[2] These limitations produce an isoquant and isocline map as illustrated in Figure 8 which will be comparable to Figure 5 with the previously outlined limitations.[3]

In Figure 8 the line EF represents the stomach capacity line imposed by the size of the rumen and the filling capacity of the ration components. Line AF represents the physiological minimum amount of hay that should be fed to allow for proper functioning of the rumen. If the

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1 M. E. Campling, M. Freer, and C.C. Balch. "Factors affecting the voluntary intake of food by cows," British Journal of Nutrition, XVI (1962), 115 - 224.

2 W. Holmes, G.W. Arnold, and A.L. Provan. "Bulk Feeds for Milk Production. I. The Influence of Level of Concentrate Feeding in Addition to Silage and Hay on Milk Yield and Milk Composition," Journal of Dairy Research, XXVII (1960), 191 - 204.

3 A similar boundary diagram was determined by Heady and Dillon, op cit. page 103.

feed input figures are not considered as input above maintenance, a further restriction for maintenance can be included in Figure 8. If R kilograms of hay are required to provide a maintenance ration for the cow and if P kg of grain will also provide sufficient energy for maintenance, then the line RP represents the maintenance restriction on the production surface. These restrictions then leave an area of possible production within the confines of EFNR.

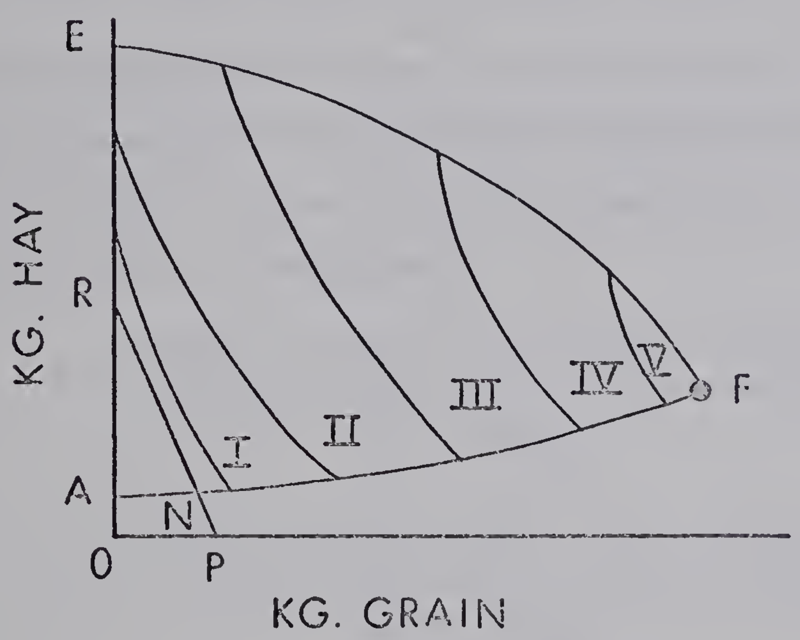

Figure 8

ISOQUANTS AND ISOCLINES RESTRICTED BY THE LIMITS OF THE COW AS PRODUCTIVE UNIT

Maximizing Returns

From the previous analysis of the production function (equation 3) the level of output can be defined and the most efficient means of producing this output, in terms of resource use, determined. However,

does the level of output specified in the previous analysis as maximum necessarily result in the maximization of returns above costs? In order to maximize the returns, the marginal value product of grain, MVP_G, must equal the cost of the grain, P_G. The MVP_G is

$$(14) \quad MVP_G = P_m a_3 - P_m 2a_4 G \pm P_m a_5 H$$

determined by multiplying the MPP_G (equation 6) by the price of milk, P_m. Then in order to maximize returns

$$(15) \quad P_G = MVP_G = P_m a_3 - P_m 2a_4 G \pm P_m a_5 H$$

and using a similar analysis for hay

$$(16) \quad P_H = MVP_H = P_m a_1 - P_m 2a_2 H \pm P_m a_5 G$$

Solving equations 15 and 16 simultaneously, the quantities of hay and grain (equation 17) determined represent these quantities at which $MVP_G = P_G$ and $MVP_H = P_H$ and thus the quantities of the feed components at which profit is maximized.

$$(17) \quad H = \frac{(2a_4 P_H + a_5 P_G)\, P_m^{-1} - a_3 a_5 - 2a_1 a_4}{a_5^2 - 4a_2 a_4}$$

$$G = \frac{(2a_2 P_G + a_5 P_H)\, P_m^{-1} - a_1 a_5 - 2a_2 a_3}{a_5^2 - 4a_2 a_4}$$

The previous analysis had all been conducted as though there existed an unlimited availability and unrestricted use of the resources in the production of milk. Equations 15 and 16 can set up in such a manner to allow both to equal 1 (equation 18) under the previously outlined

$$(18) \quad \frac{MVP_H}{P_H} = \frac{MVP_G}{P_G} = 1.0$$

conditions. However, if resource availability or resource use were limited, profit maximization would occur when due to the inability

$$(19) \quad \frac{MVP_H}{P_H} = \frac{MVP_G}{P_G} > 1.0$$

to equate the marginal value product and price.

The optimum quantity of each resource that should be used then is a function of

1) its marginal physical product
2) its price
3) the price of the product
4) the availability of the resources
5) its unlimited economic use in production of the product.

The quantity of the resource used will vary directly with the price of the product and indirectly with its own price.

This analysis makes no reference to fixed costs nor are they in any way involved in the determination of the input levels of the feed components or the resulting profit maximizing output of milk. If fixed costs of feeding were to be included, it could be done by altering the price of the hay and grain sufficiently to cover the cost of feeding a unit of the resource under the method which is to be used.

CHAPTER III

THE EXPERIMENT

Design

A 3 x 3 factorial design was used to provide a checkerboard effect of the three levels of hay and their supplementation with a grain mixture to provide three levels of digestible energy intake by the cows. The varying combinations of hay and grain provided alternate rations that supplied the recommended quantity of energy to the cow. Statistically the design of the experiment provided a means of separating the effect of the various levels of hay from any influence of the variation in the level of grain fed. Similarly the effect of the varying levels of grain can be separated from the effects of the level of hay fed. The method of separation of the variation attributable to each resource was outlined by Ostle through the use of an analysis of variance table. [1]

Level of Feeding

The three levels of hay provided were

1) 0.75 kg/100kg liveweight
2) 1.50 kg/100 liveweight
3) 2.50 kg/100kg liveweight

Using these three levels of hay, it was hoped to provide a range of consumption from near the minimum that a cow requires for normal functioning of the rumen to a maximum that the cow may consume. Each of these groups were then divided into three, with each sub-group fed at different levels of digestible energy intake. The digestible energy intake was based on the requirements outlined by the National Research Council[2] and consisted of supplementation of the level of hay with a grain mixture to provide

1) 90% National Research Council digestible energy requirements
2) 100% National Research Council digestible energy requirements
3) 120% National Research Council digestible energy requirements

The digestible energy requirements were determined by computing the

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1 B. Ostle, Statistics in Research, 2nd edition (Ames, Iowa: Iowa State University Press, 1963), pp. 164-166.

2 National Academy of Sciences - National Research Council, op. cit., pp. 2-6

digestible energy required by the cow for maintenance of its body weight and adding this amount to the digestible energy required to support the cow's present level of milk production as recommended by the National Research Council. The quantity of digestible energy determined was then corrected to one of the levels of energy depending upon the group to which the given cow had been assigned. The digestible energy provided by the hay alloted to the cow was supplemented by sufficient grain to equate the digestible energy level provided by the hay and grain with the quantity of corrected digestible energy which was to be provided.

The Cows

The 27 cows available for the experiment were divided into three groups as determined by the past production records of each cow. The first group consisted of the nine cows expected to be the best producers from the 18 to be assigned to the high or low production group. The remaining nine cows then made up the second group of expected low producers. The third group consisted of cows on their first lactation for which no estimate of production was available. One cow from each of these outlined groups was then randomly allocated to one of the cells of the basic design providing one cow of expected high production, one cow of expected low production, and one cow on its first lactation in each of the cells (Table 1).

Table 1

ALLOTMENT OF COWS BY IDENTIFICATION NUMBER TO THE BASIC EXPERIMENT

Energy Group	120% Energy			100% Energy			90% Energy		
Kg hay/100kg liveweight	.75	1.5	2.5	.75	1.5	2.5	.75	1.5	2.5
Expected high producers	809	237	233	232	124	127	707	913	022
1st calf heifers	403	410	451	503	504	506	507	501	408
Expected low producers	343	347	348	345	342	130	235	810	344

The Feed

The hay used was field cured alfalfa-bromegrass hay with crude protein content which ranged from 13 to 17 percent. In determining the digestible energy content of the hay 2.10M cal. [1] of digestible energy per kg was

1 The digestible energy content of hay was based on calculation from the National Research Council Nutrient Requirements of Dairy Cattle p 27

used as a constant factor for the conversion to an energy basis throughout the experiment. The grain mixture (Table 2) was based on an aggregate crude protein content of fourteen percent and supplemented to possess a vitamin content of 4,500 International Units of vitamin A and 950 International Units of vitamin D2 per kg of concentrate. In converting the grain mixture to energy values a conversion factor of 3.08M cal.[1] of digestible energy per kg of grain mixture was used.

Table 2

COMPOSITION OF THE GRAIN MIXTURE

Component	Kg.
Rolled barley	560
Rolled oats	250
Soybean meal	105
Wheat bran	30
Dried molasses	20
Sodium-tripolyphosphate	20
Cobalt-iodized salt	20
Vitamin premix[a]	5
	1000

[a] Vitamin premix: soybean oil meal, 3.6 kg; vit. A (10,000I.U./g). 360g; vit D_2 (35,000I.U./g), 22g.

Feeding Practises

The first cow was started on the experiment during May 1966 and the last was started during March 1967. Three weeks prior to expected calving the cows were placed on a standard ration of hay on a free feed basis. The grain intake was gradually increased until the cows were consuming all they wanted during the last week prior to calving. Following calving the daily ration was adjusted to fit the requirements

1 The digestible energy valve for the grain was calculated as an average of the components based on the National Research Council Nutrient Requirements of Dairy Cattle p27-32

of the group to which the cow had been assigned. The allotted quantities of hay and grain were weighed and fed twice daily with any residue of both removed and weighed prior to the next feeding. The amount of hay to be fed was adjusted every two weeks to account for the body weight changes of the cow. The grain fed was altered every week on the basis of the amount of milk produced during the previous week. Body weight changes of the cow were determined every two weeks.

Determination of the Data

The amounts of hay and grain consumed by each cow were recorded on a daily basis and totaled weekly. A similar procedure was followed for each cow's milk production. Milk samples were taken from two successive milkings twice monthly to ninety days and then once monthly to the end of the lactation. These samples were then analysed for butterfat, solids not fat, and crude protein. All milk weights were then converted to a mature equivalent using the correction factors of Kendrick[1] as outlined in Table 3. The mature equivalent milk data was then corrected to an energy base using both the four percent fat-corrected milk equation of Gaines[2] and the solids-corrected milk equation of Tyrell & Reid[3].

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1 J.F. Kendrick, Standardizing and Improving Dairy Herd Improvement Association Records in Proving Sires, A.R.S.-52-1 (Washington : U.S.D.A.; 1955).

2 W.L. Gaines, The Energy of Measuring Milk Yield in Dairy Cows, Bulletin 308 (Urbana, Illinois, Agr. Exp. Station, 1928).

3 H.F. Tyrell and J.T. Reid "Prediction of the Energy Value of Cow's Milk," Journal of Dairy Science, XLV133 (1965), 1215-1223.

Table 3

FACTORS USED TO CORRECT MILK PRODUCTION TO MATURE EQUIVALENT FOR THE RESPECTIVE COWS IN THE ALLOTMENT OF TABLE 1.

Energy Level	120% Energy			100% Energy			90% Energy		
Kg hay/100kg liveweight	3/4-1	1/2-2	1/2	3/4-1	1/2-2	1/2	3/4-1	1/2-2	1/2
Expected high producers	1	1.09	1.03	1.03	1.01	1.02	1	1	1.01
1st calf heifers	1.29	1.30	1.25	1.33	1.31	1.33	1.31	1.29	1.26
Expected low producers	1.15	1.17	1.17	1.17	1.16	1.10	1.40	1	1.14

The second method of correction also accounts for the solids not fat content of the milk and as outlined by Tyrell & Reid.[1] Their article gives a more accurate estimate of the energy value of milk at the extremes of possible fat content than does the four percent fat-corrected milk equation. By not compensating for the energy provided by the solids not fat separately, the fat-corrected milk equation tended to underestimate the energy content of low fat percentage milk and overestimate the energy level of milk with high fat percentages. The two correction equations as used were:

$$FCM = .4\ M + 15\ F$$

$$SCM = 12.3\ (F) + 6.56\ (SNF) - .0752\ (M)$$

Where

FCM = kilograms of four percent fat-corrected milk

SCM = kilograms of solids-corrected milk.

F = kilograms fat produced = kilograms x percent fat.

SNF = kilograms solids not fat produced = kilograms x percent solids not fat.

M = kilograms mature equivalent milk produced.

The major analysis was carried out with fat-corrected milk as the dependent variable. Fat-corrected milk corrects the production to a base similar to those reported in other studies. Canadian farmers are paid only on the basis of the actual weight and fat content of their milk, so fat-corrected milk more nearly approaches the practical method of milk correction than does solids-corrected milk which also corrects for content of solids

1 Ibid

not fat. The solids-corrected milk provides a new method of approach and may be more significant in the future as the importance of milk fat declines.

CHAPTER IV

PERSISTENCY, TOTAL PRODUCTION, AND WEIGHT CHANGES

Persistency of Production

The major problem which arose in connection with the data was uniformity in lactation length. The experiment was designed to obtain weekly records covering the 44-week period (308 days) of a normal lactation. However, many of the cows did not produce for the full 44 weeks; they were removed from the experiment at a time when production reached a low level that did not warrant the continuation of the cow on the test. Table 4 outlines the number of the week and the corresponding number of cows that produced records for those weeks. Placing the 11 cows to complete 44 weekly records and the 18 cows to finish 37 weeks into two groups based on (1) expected production, (2) level of hay feeding, and (3) National Research Council energy level indicated any variation in the number of cows reaching 44 and 37 weeks respectively in each of the sub groups. This method of examination produced no pattern of persistency in the expected production or in the hay groups; however, in the energy groups there was a reduction in the number of cows finishing the required time as the energy level was reduced. Classification as given in Table 5 allows no means of compensation for the relative position in which the cows finish nor does this method of examination allow for compensation due to the relavent ability of a cow to produce.

One of the factors which affects the persistency of production is the level of nutrition that the cow receives.[1] Another factor

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1 Shown by A.W.A. Burt, "The Influence of Level of Feeding During Lactation Upon the Yield and Composition of Milk", Dairy Science Abstracts XIX (1957), 435-453; M.E. Castle and J.N. Watson. "The Effect of Level of Concentrate Feeding Before and After Calving on The Production of Dairy Cows," Journal of Dairy Research, XXVIII (1961), 231-243; and D.K. Hotchkiss, N.L. Jacobson, and C.P. Cox. "Effect of Various Hay : Concentrate Ratios and Levels of Feeding on Production and Composition of Milk," Journal of Dairy Science, XLIII (1960), 872 abstract.

Table 4

THE NUMBER OF THE WEEK AND THE CORRESPONDING NUMBER OF COWS WHICH PRODUCED RECORDS FOR THE WEEK

Week or week inclusive	Number of cows recording for the week(s)
1 to 20	27
21 to 25	26
26 to 28	25
29 to 30	24
31	23
32 and 33	22
34	21
35	20
36	19
37	18
38 to 41	15
42	14
43 and 44	11

Table 5

DISTRIBUTION OF THE 11 COWS THAT FINISHED 44 WEEKS AND THE 18 THAT FINISHED 37 WEEKS IN EACH OF THE MAJOR GROUPINGS.

Weeks completed	Group Indicating expected production			Kg of Hay/100 body weight			Percent of NRC energy recommendations		
	high	low	heifers	.75	1.5	2.5	90	100	120
44	3	3	5	4	3	4	0	5	6
37	7	3	8	7	7	4	3	7	8

which affects the length of a lactation is the cow's inherited genetic ability to produce as shown by Lasley[1] in his heritability table for dairy cow characteristics Comparing the performance of the cows during the previous lactation with the cow's performance during the experimental lactation (Table 6) gave an indication of the effect of the experiment and any bias that resulted from the cow's inherent ability to produce. Given the allotment of the cows to the experiment on the basis of their expected production, a reduction of 5.6 weeks occurred in the expected low producers. A partial explanation is that the high producing cows on a low energy ration suffered from a lack of energy causing a reduction in production that would not have occurred during the previous lactation. The heifers had a longer lactation average than either of the groups of cows. The longer lactation could be attributed to a lower level of production from the heifers.

Table 6

DIFFERENCES IN AVERAGE LACTATION LENGTH FOR THE COWS BEFORE AND WHILE ON THE EXPERIMENT

cows and lactation	Expected level of production			Kg hay per 100 kg Body weight			Percent NRC recommendations		
	high	low	heifers	.75	1.5	2.5	90	100	120
Cows with previous lactation	44.0	37.3	-	43.6	44.0	36.5	39.8	44.0	40.3
Above cows while on experiment	38.4	36.4	-	39.4	38.6	35.0	32.8	40.5	40.1
All cows on experiment	38.4	36.4	40.0	40.3	39.2	35.3	33.3	40.9	40.6

1 J.F. Lasley, Genetics of Livestock Improvement (Englewood Cliffs, New Jersey: Prentice-Hall Inc., 1963), p.305.

During the previous lactation the cows as classified in the hay groups produced almost to the 44-week limit except for the cows in the 2.5 kilogram group, which completed only 36.5 weeks. Even with the reduction in lactation length for the .75 and 1.5 kilograms of hay groups the lactation length was reduced as the hay fed increased. The reduction in lactation length with increased hay consumption was still noticed when the longer lactations of the heifers were included. The heifers add .9, .6, and .3 weeks to the average lactation length of the .75, 1.5, and 2.5 kilograms of hay groups, respectively, indicating a declining influence of the heifers as the quantity of hay offered is increased.

The largest variation among groups occurred in the energy groups. According to the records of the previous lactation the average lactation length would have been about 40 weeks for the 90 and 120 percent energy groups and near 44 weeks for the 100 percent group. With the cows having previous records the 90 percent group lost seven weeks, the 100 percent group lost three and one half weeks and the 120 percent group lost two tenths of a week. The change in the 120 percent group may be attributed to chance variation, while the change in the 100 percent group may be the result of chance variation and also the effect of the varying hay levels which were not present in the previous lactation. The large reduction of lactation period in the 90 percent group was much greater than the change among any of the hay groups or expected production groups; therefore, much of the change was attributed to the lower energy level which the cows received. Inclusion of the heifers resulted in an increase of .5, .7, and .5 weeks for the 90, 100, and 120 percent energy groups, respectively. The 100 percent group had a longer average lactation than did the 120 percent group both on the experiment and in the previous lactation so the reversal of the two groups could be attributed to the ability of the cows.

Using the analysis on the basis of average lactation length as the criterion for selecting a ration to feed cows would result in the use of the 100 percent level of National Research Council recommendations and the .75 kilograms of hay per 100 kilograms of body weight. The lactation length was affected by the magnitude of the maximum point of production

and the average weekly decline in the production from this maximum.

The average rate of decline in production was estimated by the linear regression technique. The average weekly production of milk, fat-corrected milk, and solids-corrected milk were computed for each of the three levels of energy. The average weekly production of the group was used as the dependent variable with time as the independent variable. The determined coefficient for time expressed the average rate of decline of weekly production. The intercepts followed a similar pattern for each of the dependent variables (Table 7). The 100 percent energy group had the largest intercept, then the 90 percent energy group followed by the 120 percent energy group with the lowest intercept. The coefficient for the time variable of the 90 percent energy group was greater in absolute value than the coefficient for either the 100 or 120 percent energy groups. The difference between the 100 and 120 percent energy groups followed no definite pattern (the differences, 120 percent - 100 percent, were .05, - .03, and - .11) with the 100 percent group being the largest on two of the occasions. The production in week 44 was predicted by the equation, and the position of the energy groups changed with the 100 percent groups still being the largest but now followed by the 120 percent group and then the 90 percent group. The respective position and rate of decline of each of the energy groups with respect to each other are illustrated in figures 9, 10, and 11 for the milk, fat-corrected milk, and solids-corrected milk analysis, respectively.

Correcting the production of mature equivalent milk to a constant energy base of either fat-corrected milk or solids-corrected milk resulted in a reduction of the height of the intercepts and an accompanying reduction in the rate of decline represented by the slope. The correction resulted in a more horizontal regression line for the fat-corrected milk than the regression line determined for milk and an intermediate position for the solids-corrected milk regression.

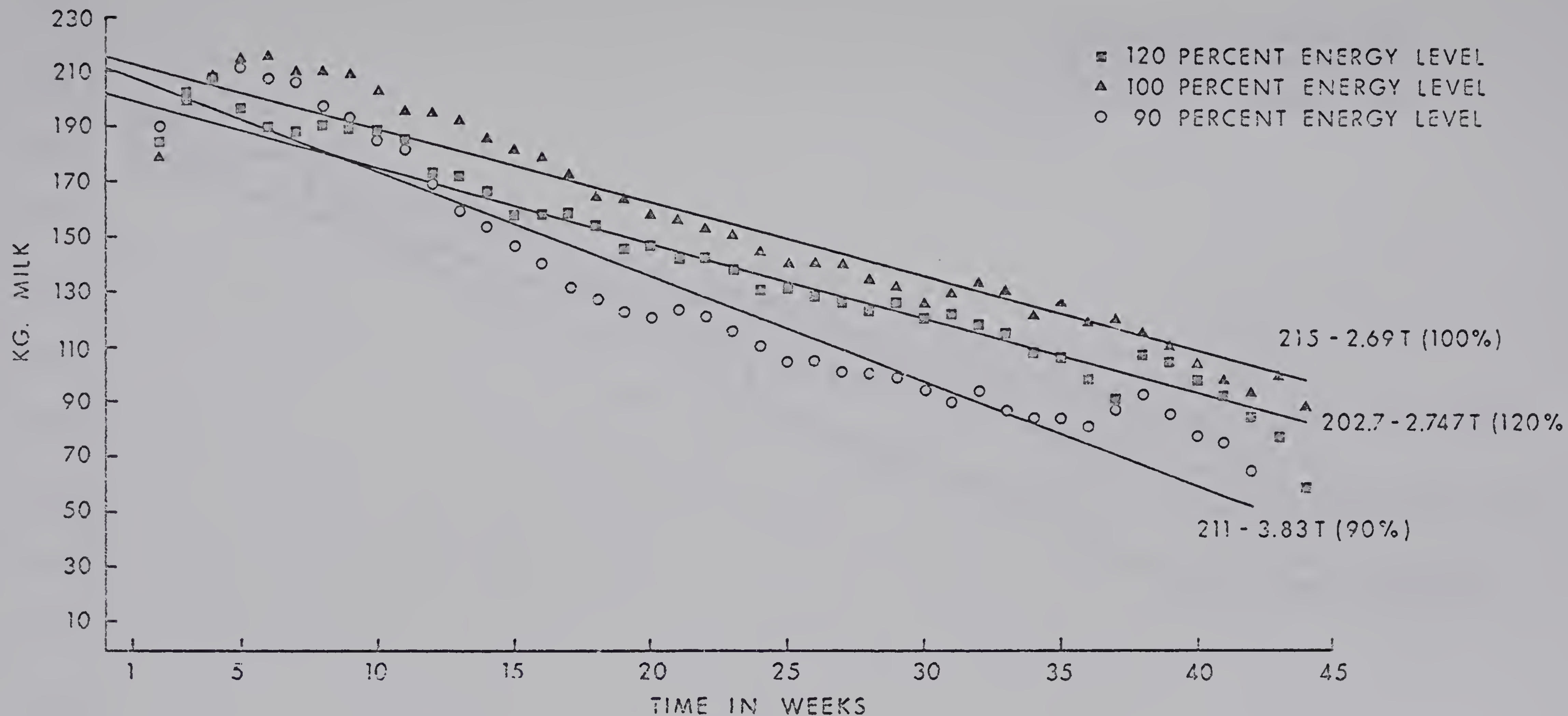

Figure 9

AVERAGE WEEKLY PRODUCTION OF UNCORRECTED MILK AND THE REGRESSION LINE FOR THE NINE COWS ON EACH OF THE THREE ENERGY LEVELS

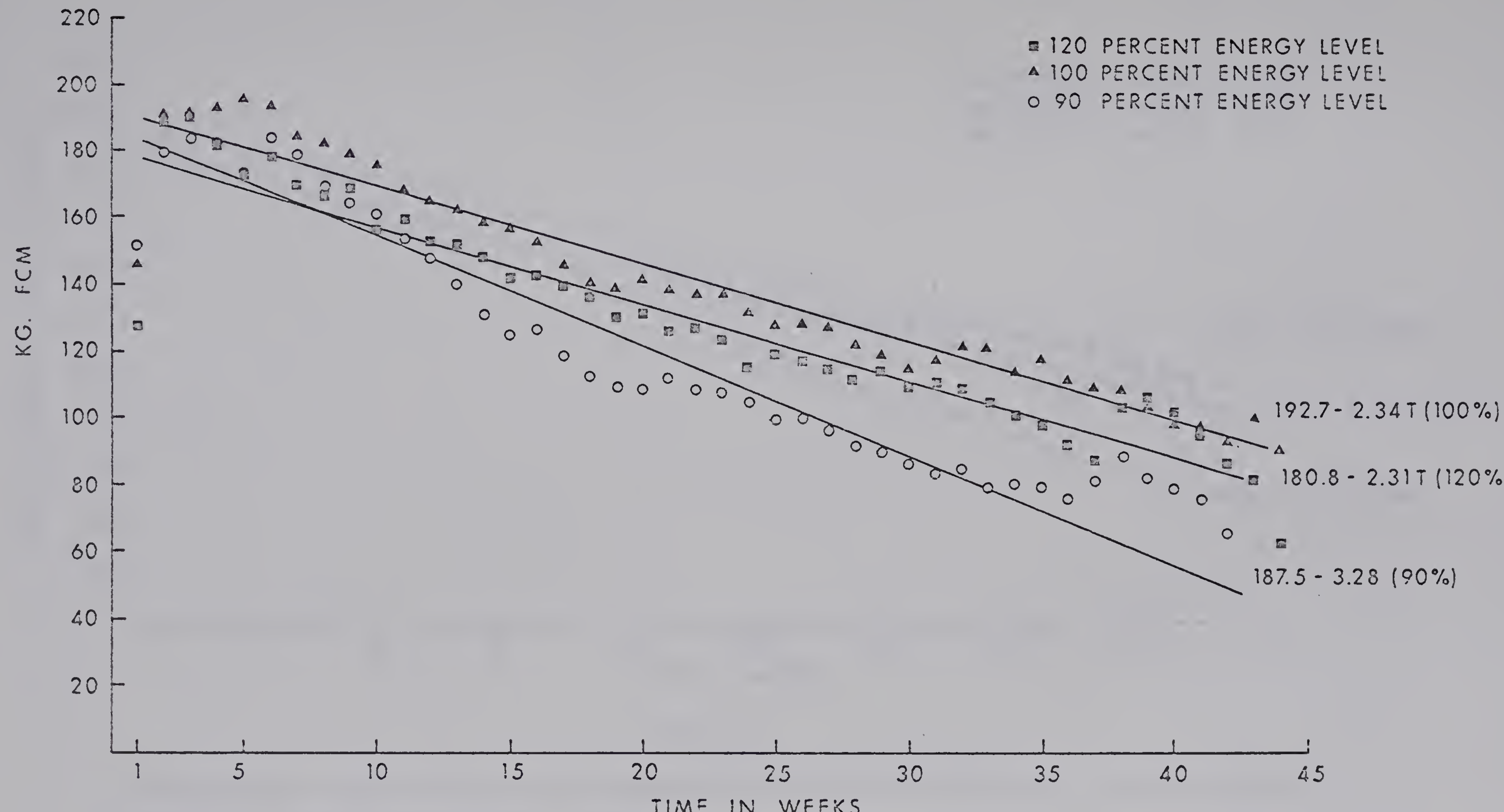

Figure 10

AVERAGE WEEKLY PRODUCTION OF FAT-CORRECTED MILK AND THE REGRESSION LINE FOR THE NINE COWS ON EACH OF THE THREE ENERGY LEVELS

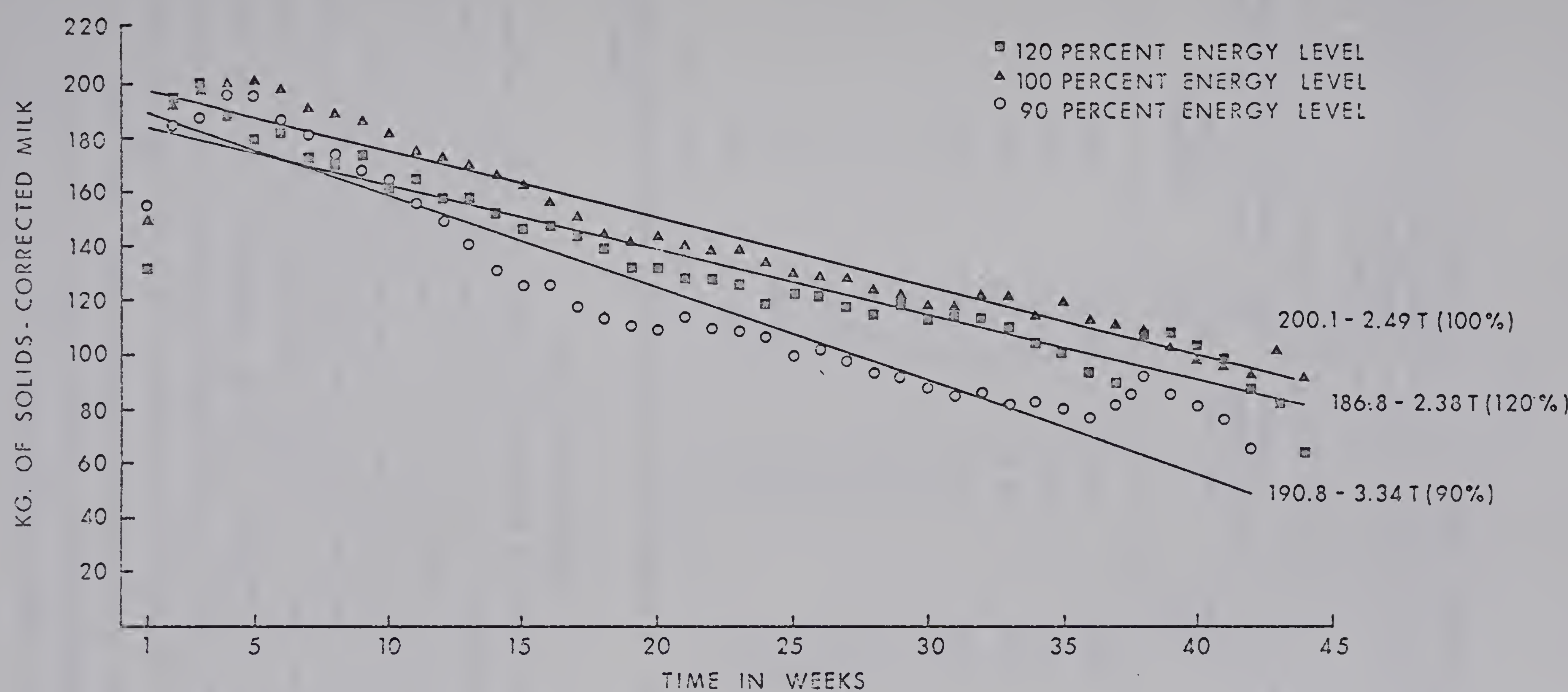

Figure 11

AVERAGE WEEKLY PRODUCTION OF SOLIDS-CORRECTED MILK AND THE REGRESSION LINE FOR THE NINE COWS ON EACH OF THE THREE ENERGY LEVELS

Table 7

LINEAR REGRESSION DATA FOR EACH OF THE ENERGY GROUPS WITH AVERAGE WEEKLY PRODUCTION OF MILK, FAT-CORRECTED MILK, AND SOLIDS-CORRECTED MILK TAKEN INDIVIDUALLY AS THE DEPENDENT VARIABLES AND THE PREDICTED QUANTITY OF AVERAGE WEEKLY PRODUCTION DURING WEEK 44 IN EACH GROUP.

Dependent variable	Energy group as a percent of NRC standards	Value of inter-cept	Linear coefficient for time	"t" value for time coeffi-cient *	r^2	Predicted production in week 44 (kilograms)
Milk	120	202.69	-2.74	16.87	.871	82.13
	100	215.30	-2.69	14.35	.831	96.94
	90	211.74	-3.83	18.41	.890	43.22
FCM	120	180.83	-2.31	18.15	.887	79.19
	100	192.71	-2.34	21.71	.918	89.75
	90	187.46	-3.28	17.36	.878	43.14
SCM	120	186.80	-2.38	18.06	.886	82.08
	100	200.12	-2.49	21.37	.916	90.56
	90	190.79	-3.34	17.10	.874	43.83

* t value greater than 2.576 gives significance to the coefficient at the 99% level.

Total production

Another aspect of production related to the area of lactaton length and position of the maximum point and rate of decline of the lactation curve was total production. Total production of the cows was based on mature equivalent four percent fat-corrected milk and was measured over the entire lactation of each cow as well as over 37 weeks for the 18 cows that completed these weeks (Table 8). The average production was 5356 kg, 5834 kilograms, and 4282 kilograms for the 120 percent, 100 percent, and 90 percent energy groups, respectively, with an average of 5160 kilograms for all the cows on the experiment. Total production was at a lower level than expected with only nine cows producing over 6,000 kilograms, one of which was over 7,000 kilograms while there were six that failed to reach the 4,000 kilogram level with two of these below 3,000 kilograms.

Table 8

TOTAL PRODUCTION OF MATURE EQUIVALENT FOUR PERCENT FAT-CORRECTED MILK ON THE BASIS OF ENERGY GROUPS OVER THE TOTAL LACTATION FOR ALL COWS AND FOR 37 WEEKS FOR 18 COWS.

120 percent group			100percent group			90 percent group		
Cow no.	Lactation total kg	37 week total kg	Cow no.	Lactation total kg	37 week kg.	Cow no.	Lactation total kg	37 week total kg.
809	6436(44)	5906	503	6905(44)	5960	501	6070(42)	5656
343	5371(44)	4896	345	6445(44)	5717	507	4116(41)	3813
410	6447(44)	5578	504	6577(44)	5804	913	5286(37)	5286
347	5842(44)	5302	127	7883(44)	7164	235	4055(36)	*
233	6741(44)	6041	506	5925(44)	5306	344	3747(35)	*
451	5390(44)	4671	124	4976(42)	4608	810	5132(33)	*
237	5204(37)	5204	232	5093(42)	4787	707	4533(31)	*
403	3406(37)	3406	130	5907(34)	*	022	3507(25)	*
348	3364(28)	*	342	2873(30)	*	408	2093(20)	*

* These cows failed to produce for 37 weeks

() Represent weeks the cow produced.

Weight Change of the Cows

The loss of weight by the dairy cow during the initial part of the lactation is an accepted occurrence by most dairy nutritionists. The cow should be fed at a level of nutrition sufficient to enable her to replace the lost weight during the latter part of the lactation. Flatt[1] indicated that no detrimental effect occurred when cows lose weight in the initial part of the lactation because it is difficult for good cows to consume the required amount of energy in any form to sustain the level of production reached by some of the cows. They also reported an increased efficiency of fat deposition by the lactating dairy cow as opposed to the non-lactating dairy cow.

1 W.P. Flatt, L.A. Moore, N.W. Hoover, and R.D. Plowman, "Energy Metabolism Studies with a High Producing Lactating Cow," Journal of Dairy Science , XLVIII (1965), 797-798

An examination of the cows' weights during the experiment (Table 24) reveals that, of the cows on the experiment, cow 451, a first calf heifer in the 2.5 kilograms of hay per 100 kg body weight group supplemented to the 120 percent energy level, was the only cow to gain weight throughout the lactation. Cows 130 and 348 had their lowest weights at the end of the lactation but both had periods of weight increase after an initial decline, then a final decline to the lightest weight of their experimental lactation. A weight analysis on the basis of a net weight change (Table 25), reveals that 14 cows had a net weight gain, a positive sum of the inter-period weight change, while 13 had net weight losses. Adding the net weight changes, gave an overall average body weight change per cow of -13.1 kilograms with a standard deviation of 80.53. The range of net changes was from a net gain in weight of 127 kilograms (cow 451) to a net loss of 194 kilograms (cow 127).

Within the energy groups the weight changes averaged 38 kilograms, -29 kilograms, and -48 kilograms for the 120 percent, 100 percent, and 90 percent groups, respectively, with corresponding standard deviations of 49.27, 84.92, and 84.85. The cows on the 90 percent energy rations had a net loss in weight that was greater than the loss of the cows on the 100 percent level; both were losses in weight compared to the weight gains of the 120 percent group. When the weight losses are aligned with the persistency figures and the milk production figures (Table 9), any advantage of the weight gain by the 120 percent group was not verified because the 100 percent group was comparable in persistency and above the 120 percent group in average production. The large loss of weight of the 90 percent energy group was a factor to consider because this group had the shortest lactation by seven weeks and the lowest total production by 1561 kilograms.

Table 9

COMPARISON OF AVERAGE LACTATION LENGTH, AVERAGE TOTAL PRODUCTION AND AVERAGE WEIGHT CHANGE OF THE COWS IN THE THREE ENERGY GROUPS.

Energy group	Average lactation length (weeks)	Average total production (kilograms)	Average weight change (kilograms)
120 percent	40.6	5356	38
100 percent	40.9	5843	-29
90 percent	33.3	4282	-48

CHAPTER V

PRODUCTION FUNCTION DETERMINATION AND MARGINAL ANALYSIS

Production Functions with Fat-Corrected Milk Dependent

The initial regression analysis was performed on the individual weekly records of all the cows completing records each week (Table 26). Of the weekly equations, only 18 of the possible 44 were of the form that a positive sign was present on both linear terms and a negative sign was present on both squared terms. Equations with these signs on their coefficients were consistent with maximization of a function. In order to reduce the number of equations, the weekly data was combined into periods of four weeks. This procedure allowed for the continued use of only one time period in each equation. The four week groups approximated monthly periods that, on a practical level, would approach the time periods during which a farmer might change the cow's ration on the basis of monthly milk recording systems.

Rather than include all the cows producing records during each period, only the 18 cows that produced at least 37 weeks were used. In this manner each period had the same cows and the inter-period comparisons contained no bias due to a different number of cows being used in each period. Of the eighteen cows used in the grouped analysis, eight were from the 120 percent energy group, seven were from the 100 percent energy group, and three were from the 90 percent energy group. To expand the analysis to include the four-week period, 38-41, would have resulted in a reduction in cow numbers to 15 and little gain from the added period, for the production of the cows was falling rapidly and little could be done to bring forth any change in the production level at a position as late in the lactation as weeks, 38-41. The periods as they were formed for the grouped analysis were weeks 2 to 5, 6 to 9, 10 to 13, 14 to 17, 18 to 21, 22 to 25, 26 to 29, 30 to 33, and 34 to 37, inclusive. The quadratic regression with the independent variables based on hay and grain consumption including a term expressing the interaction of hay and grain were then determined for the totals in each period. Periods 2 to 5, 14 to 17, 18 to 21, 22 to 25, 26 to 29, and 34 to 37 gave equations that indicated a maximizing function (Table 10). However

Table 10

REGRESSION EQUATIONS FOR GROUPED RECORDS WITH FCM DEPENDENT; USING THE 18 COWS COMPLETING AT LEAST 37 WEEKS

Period	Equation No.	Equation	R^2	F Value	Standard Error Of Estimating
2 - 5	22	$FCM = -165.03 + \underset{(1.477)^*}{4.3859H} + \underset{(.533)}{2.0793G} - \underset{(.790)}{.0058H^2} - \underset{(.186)}{.0010G^2} - \underset{(.493)}{.0036HG}$	.531	2.713	127.02
6 - 9	23	$FCM = 309.53 + \underset{(1.213)}{3.1563H} - \underset{(.388)}{1.0525G} - \underset{(1.076)}{.0077H^2} + \underset{(.605)}{.0026G^2} + \underset{(.536)}{.0026HG}$	.669	4.852	103.82
10 - 13	24	$FCM = 33.41 + \underset{(.209)}{.6048H} + \underset{(.666)}{1.8066G} + \underset{(.400)}{.0022H^2} - \underset{(.231)}{.0008G^2} + \underset{(.441)}{.0018HG}$	.581	3.332	91.56
14 - 17	25	$FCM = -87.67 + \underset{(.642)}{1.5919H} + \underset{(.759)}{2.0421G} - \underset{(.061)}{.0003H^2} - \underset{(.373)}{.0012G^2} - \underset{(.515)}{.0024HG}$	.505	2.452	88.09
18 - 21	26	$FCM = -370.73 + \underset{(1.165)}{3.0453H} + \underset{(.765)}{3.1276G} - \underset{(.536)}{.0020H^2} - \underset{(.439)}{.0021G^2} - \underset{(.906)}{.0057HG}$	.434	1.841	78.39
22 - 25	27	$FCM = -423.48 + \underset{(1.434)}{4.2099H} + \underset{(.528)}{2.7120G} - \underset{(1.275)}{.0043H^2} - \underset{(.171)}{.0011G^2} - \underset{(.883)}{.0069HG}$	.409	1.658	76.82
26 - 29	28	$FCM = -1154.33 + \underset{(1.542)}{5.2349H} + \underset{(1.464)}{7.4359G} - \underset{(1.025)}{.0035H^2} - \underset{(1.192)}{.0076G^2} - \underset{(1.516)}{.0131HG}$	.422	1.755	65.91
30 - 33	29	$FCM = 699.37 - \underset{(.328)}{1.8162H} - \underset{(.355)}{2.7859G} + \underset{(.539)}{.0032H^2} + \underset{(.537)}{.0058G^2} + \underset{(4.09)}{.0058HG}$	.346	1.270	88.90
34 - 37	30	$FCM = -794.62 + \underset{(1.145)}{3.4731H} + \underset{(1.528)}{5.7819G} - \underset{(.458)}{.0016H^2} - \underset{(1.080)}{.0059G^2} - \underset{(1.232)}{.0095HG}$	.750	7.186	63.43

* These numbers represent the "t" values for the coefficients directly above them

in these equations the statistical significance of the coefficients was low. Six of a possible 30 of the coefficients in the six maximizing equations reached the 90 percent level of significance and an additional eleven reached the 80 percent level of significance. Five of the equations had coefficients for the linear hay term significant at the 80 percent level, while the linear grain term, the hay squared term, and the grain squared term were each significant to the 80 percent level in two of the six equations. The hay-grain term was significant to the 80 percent level in four of the equations. On the basis of the individual equations, the number of coefficients significant to the 80 percent level were 0, 1, 2, 3, 4, and 5 for equations 28, 30, 27, 26, 22, and 25, respectively.

In an attempt to increase the level of significance of the equations and also to increase the explanatory power of the grouped data equations, the body weight change of the cow during the four-week period was included as a linear independent variable. The weight losses during the initial weeks of the lactation were considered as positive inputs, while the weight gains of the latter part of the lactation were considered to be negative inputs. The analysis resulted in a set of equations (Table 27) six of which gave maximizing functions and corresponded to the same periods as the six equations in the initial analysis. The explanatory power of equations 25A, 27A, and 28A, increased by .014, .025, and .122, respectively. In the other equations the increase was less than .01. Only in equation 28A did the weight change term ontain a coefficient significant to the 90 percent level. None of the weight change coefficients of the other equations attained significance at the .75 percent level. This additional variable produced little change in the significance of the existing coefficients and also did not produce any consistent increase in the percentage of the variation explained by the equation. The original set of equations (Table 10) were then used for the marginal analysis.

The coefficients of the terms in the equations of Table 10 should have followed a pattern consistent with the response expected from a cow fed the feed components. The linear coefficient on the grain term should have been larger than the linear term for hay, as the cow was

expected to respond with a greater increase in production with an extra unit of grain than an extra input of hay. The larger coefficient on the linear grain term should have corresponded to a larger coefficient on the grain squared term than was on the hay squared term. Of the six equations having the desired signs on the coefficients, equations 25, 26, 28, and 30 had larger grain and grain squared coefficients than the corresponding hay and hay squared coefficients. Equations 22 and 27 had larger hay coefficients than grain coefficients coupled with larger hay squared coefficients than grain squared coefficients. The difference between the equations did not carry into the isoquant analysis where equation 27 became similar to equations 25, 26, 28, and 30 in that they all produced isoquants which were concave to the origin and equation 22 was the only equation which produced isoquants that were convex to the origin.

Isoquant Determination and Marginal Analysis

In the analysis of the milk production functions isoquants convex to the origin were desired (Figure 5). Convex isoquants indicate diminishing marginal rate of feed component substitution; that is, to replace a successive unit of grain with hay, more hay must be offered than was required to replace the preceeding unit of grain. In the marginal analysis as the quantity of hay increased, the marginal rate of transformation of hay into milk should have decreased, and the marginal rate of substitution of hay for grain should have increased. Similarly, as the quantity of grain fed increased, the marginal rate of transformation of grain into milk should have decreased, and the marginal rate of substitution of hay for grain should have decreased. As the level of milk output was increased, the marginal rates of transformation of hay or grain into milk should have decreased in accordance with the diminishing marginal productivity of each of the feed components. The pattern of changes just outlined occurred only in equation 22. The response of the marginal quantities in the other equations was varied and resulted in isoquants concave to the origin. Tables 11 to 16 give the quantities of grain that were to be combined with the given levels of hay to produce the specified quantities of milk for each of the equations 22, 25, 26, 27, 28, and 30, respectively. The hay-grain

Table 11

ANALYSIS OF EQUATION 22, PERIOD 2 TO 5 GIVING MILK ISOQUANTS, MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

	Section (a) isoquant determination[1]							Section (b) $\frac{\partial M}{\partial H}$ [2]						
Kg Hay	Kg FCM 100	200	300	400	500	600	700	100	200	300	400	500	600	700
50	32	88	149	214	285	365	458	3.69	3.49	3.27	3.04	2.78	2.49	2.16
100			51	115	185	264	355			3.04	2.81	2.56	2.27	1.95
150				25	95	174	265				2.55	2.30	2.02	1.69
200					15	95	188					2.01	1.72	1.39
250						27	124						1.39	1.04
300							77							.63
350							53							.14

	Section (c) $\frac{\partial M}{\partial G}$ [3]							Section (d) $\frac{\partial H}{\partial G}$ [4]						
Kg Hay	Kg FCM 100	200	300	400	500	600	700	100	200	300	400	500	600	700
50	1.83	1.72	1.60	1.47	1.33	1.17	.98	.50	.49	.49	.48	.48	.47	.46
100			1.62	1.49	1.35	1.19	1.01			.53	.52	.52	.52	.52
150				1.49	1.35	1.19	1.01				.58	.59	.59	.60
200					1.33	1.17	.98					.66	.66	.71
250						1.13	.93							.90
300							.84							1.35
350							.71							5.25

1 Gives the quantity of grain which in conjunction with the given level of hay will produce the specified level of milk as determined from the isoquant equations

2 $\frac{\partial M}{\partial H}$ = marginal productivity of hay, the kg of milk produced from one additional kg of hay at the given level of hay and grain in section a.

3 $\frac{\partial M}{\partial G}$ = marginal productivity of grain, the kg. of milk produced from one additional kg of grain at the given level of hay and grain in section a

4 $\frac{\partial H}{\partial G}$ = marginal substitution rate of hay for grain, the kg of hay to replace one additional kg of grain - figures given as absolute values

5 For the equations used in each section see Table 28

Table 12

ANALYSIS OF EQUATION 25, PERIOD 14 TO 17, GIVING MILK ISOQUANTS, MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

Section (a) isoquant determination a) — Section (b) $\frac{\partial M}{\partial H}$ a)

Kg FCM

Kg Hay	100	200	300	400	500	600	700	100	200	300	400	500	600	700
50	59	117	181	252	335	435	576	1.42	1.28	1.13	.96	.75	.52	.18
100	18	77	142	215	299	403	557	1.49	1.35	1.19	1.02	.81	.56	.20
150		34	100	173	259	367	533		1.42	1.66	1.09	.88	.62	.22
200			54	129	216	325	501			1.34	1.16	.95	.69	.27
250			6	81	168	278	457			1.43	1.25	1.04	.77	.34
300				29	116	225	401				1.34	1.13	.87	.45
350					59	167	333					1.64	.98	.58
400						103	257						1.10	.73
450						35	176						1.24	.89
500							91							1.07

Section (c) $\frac{\partial M}{\partial G}$ a) — Section (d) $\frac{\partial H}{\partial G}$ a)

Kg FCM

Kg Hay	100	200	300	400	500	600	700	100	200	300	400	500	600	700
50	1.78	1.64	1.49	1.32	1.12	.88	.54	1.25	1.28	1.32	1.38	1.47	1.69	3.00
100	1.76	1.62	1.46	1.29	1.08	.83	.46	1.18	1.20	1.23	1.27	1.33	1.48	2.39
150		1.60	1.44	1.27	1.06	.80	.40		1.13	1.14	1.17	1.20	1.29	1.81
200			1.43	1.25	1.04	.78	.36			1.07	1.08	1.09	1.13	1.33
250			1.43	1.25	1.04	.77	.35			1.00	1.00	1.00	1.00	1.00
300				1.25	1.04	.78	.36				.93	.92	.90	.80
350					1.05	.80	.40					.85	.82	.69
400						.83	.46						.76	.63
450						.88	.54						.71	.60
500							.62							.57

a. See Corresponding footnotes for Table 11

Table 13

ANALYSIS OF EQUATION 26, PERIOD 18 TO 21, GIVING MILK ISOQUANTS MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

	Section (a) isoquant determination [a]						Section (b) $\frac{\partial M}{\partial H}$ [a]					
Kg Hay	Kg FCM 100	200	300	400	500	600	100	200	300	400	500	600
50	125	170	220	275	340	420	2.13	1.87	1.59	1.26	.91	.45
100	78	125	177	236	306	399	2.20	1.93	1.64	1.30	.90	.37
150	27	75	129	192	269	378	2.29	2.02	1.71	1.35	.91	.29
200		21	78	143	226	356		2.12	1.80	1.43	.96	.21
250			21	89	176	334			1.93	1.54	1.04	.14
300				27	117	306				1.69	1.17	.10
350					48	233					1.37	.32
400						123						.74
450						10						1.19

	Section (c) $\frac{\partial M}{\partial G}$ [a]						Section (d) $\frac{\partial H}{\partial G}$ [a]					
Kg Hay	Kg FCM 100	200	300	400	500	600	100	200	300	400	500	600
50	2.32	2.13	1.92	1.69	1.41	1.08	1.08	1.13	1.21	1.32	1.55	2.39
100	2.23	2.03	1.82	1.57	1.27	.88	1.01	1.05	1.11	1.20	1.41	2.38
150	2.16	1.96	1.73	1.47	1.14	.69	.94	.97	1.01	1.08	1.25	2.35
200		1.90	1.66	1.39	1.04	.49		.89	.92	.97	1.08	2.25
250			1.62	1.33	.96	.30			.84	.86	.92	2.12
300				1.30	.92	.13				.77	.79	1.31
350					.93	.15					.68	.49
400						.33						.44
450						.52						.44

a) see corresponding footnotes for Table 11

Table 14

ANALYSIS OF EQUATIONS 27, PERIOD 22 TO 25, GIVING MILK ISOQUANTS MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

Section (a) isoquant determination [a] Section (b) $\frac{\partial M}{\partial H}$ [a]

Kg Hay	Kg FCM 100	200	300	400	500	100	200	300	400	500
50	147	197	250	307	369	2.77	2.42	2.05	1.66	1.23
100	75	131	191	256	328	2.83	2.44	2.03	1.58	1.08
150		55	122	197	285		2.54	2.08	1.55	.95
200			42	129	237			2.20	1.60	.85
250				42	176				1.77	.84
300					87					1.02

Section (c) $\frac{\partial M}{\partial G}$ a) Section (d) $\frac{\partial H}{\partial G}$ a)

Kg Hay	Kg FCM 100	200	300	400	500	100	200	300	400	500
50	2.04	1.93	1.82	1.69	1.56	.74	.80	.88	1.02	1.26
100	1.86	1.73	1.60	1.46	1.30	.66	.71	.79	.92	1.20
150		1.56	1.41	1.24	1.05		.61	.68	.80	1.10
200			1.24	1.04	.81			.56	.66	.95
250				.89	.59				.51	.71
300					.45					.43

a) see corresponding footnotes for Table 11

Table 15

ANALYSIS OF EQUATION 28, PERIOD 26 TO 29, GIVING MILK ISOQUANTS MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

	Section (a) isoquant determination a)					Section (b) $\frac{\partial M}{\partial H}$ a)				
Kg Hay	Kg FCM 100	200	300	400	500	100	200	300	400	500
50	187	214	244	279	325	2.44	2.09	1.69	1.22	.62
100	155	183	215	254	308	2.51	2.14	1.72	1.21	.50
150	120	149	183	226	290	2.61	2.23	1.78	1.23	.38
200	83	113	148	194	272	2.75	2.35	1.89	1.30	.28
250	43	73	109	156	247	2.92	2.52	2.05	1.44	.24
300		30	66	113	204		2.74	2.27	1.66	.46
350			19	64	141				1.95	.94
400				10	74				2.31	1.41
450					5					2.00

	Section (c) $\frac{\partial M}{\partial G}$ a)					Section (d) $\frac{\partial H}{\partial G}$ a)				
Kg Hay	Kg FCM 100	200	300	400	500	100	200	300	400	500
50	3.94	3.54	3.08	2.53	1.85	1.62	1.69	1.82	2.07	2.94
100	3.77	3.35	2.86	2.26	1.44	1.50	1.56	1.66	1.88	2.89
150	3.64	3.20	2.68	2.04	1.06	1.40	1.44	1.51	1.66	2.78
200	3.55	3.10	2.56	1.87	.69	1.29	1.32	1.35	1.44	2.48
250	3.50	3.05	2.50	1.79	.40	1.20	1.21	1.22	1.24	1.65
300		3.05	2.50	1.79	.41		1.11	1.10	1.08	.85
350			2.57	1.88	.71			1.01	.97	.75
400				2.05	1.08				.89	.73
450					1.47					.73

a See corresponding footnotes for Table 11

Table 16

ANALYSIS OF EQUATION 30, PERIOD 34 TO 37, GIVING MILK ISOQUANTS, MARGINAL PRODUCTS AND MARGINAL RATES OF SUBSTITUTION

Kg Hay	Section (a) isoquant determination a) Kg FCM 100	200	300	400	Section (b) $\frac{\partial M}{\partial H}$ a) 100	200	300	400
50	168	200	236	281	1.72	1.41	1.07	.65
100	141	174	214	263	1.82	1.50	1.12	.65
150	111	146	188	243	1.94	1.61	1.21	.69
200	77	113	157	217	2.10	1.76	1.34	.78
250	40	77	121	183	2.29	1.95	1.52	.93
300		36	80	141		2.18	1.75	1.17
350			33	91			2.04	1.49
400				35				1.87

Kg Hay	Section (c) $\frac{\partial M}{\partial G}$ a) Kg FCM 100	200	300	400	Section (d) $\frac{\partial H}{\partial G}$ a) 100	200	300	400
50	3.32	2.95	2.52	1.99	1.94	2.08	2.36	3.09
100	3.17	2.77	2.31	1.72	1.75	1.85	2.06	2.65
150	3.05	2.64	2.14	1.49	1.57	1.64	1.77	2.17
200	2.97	2.55	2.03	1.33	1.41	1.45	1.51	1.71
250	2.94	2.50	1.98	1.24	1.28	1.29	1.30	1.34
300		2.51	1.99	1.26		1.15	1.13	1.08
350			2.07	1.38			1.01	.93
400				1.57				.84

a) see corresponding footnotes for Table 11

combinations (section a of the tables) under each specified level of milk output represent the corresponding points on the isoquant for the given level of milk output.

The tables also include the marginal rates of transformation of both hay and grain into milk (section b and c of the tables) and the marginal rate of substitution of hay for grain at the given level of input (section d of the tables). Only equation 22 produced an isoquant map such that the isoquants were convex to the origin. Figures 12 to 17 represent the isoquants for the equations 22, 25, 26, 27, 28, and 30 respectively. All of the figures as well as the data in the respective tables indicate decreasing marginal rates of transformation of the two feed components into milk as the level of the respective feed ingredient was increased. Figures 12 to 17 indicate the curvature of the isoquants increased as the quantity of milk represented by the isoquant increased. The increased curvature on the isoquants representing the larger quantities of milk indicated a small variation in feed combination result in a larger variation in the rate of feed substitution than occurred for an equal variation in feed combinations at a lower level of milk output.

The 18 observations used to determine the regression equations for each period were plotted on the respective isoquant figure (Figures 12 through 17) so as to observe the dispersion of the observations over the isoquant map. The desired dispersion was one in which the observations cover a large portion of the isoquant map with increasing levels of FCM indicated as the observations proceeded away from the origin. The observations in Figure 12 followed this pattern as the observations ranged from the area of 100 Kg of hay and 200 Kg grain to 300 Kg hay and 450 Kg of grain with a reasonably even distribution below the stomach capacity line.[1] The levels of production increased as the levels of hay and grain increased in all but a few cases such as the observations of 638 and 556 representing low levels of production relative to the other observations in their vicinity. The distribution

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1 Quantities derived from the equations in Table 28

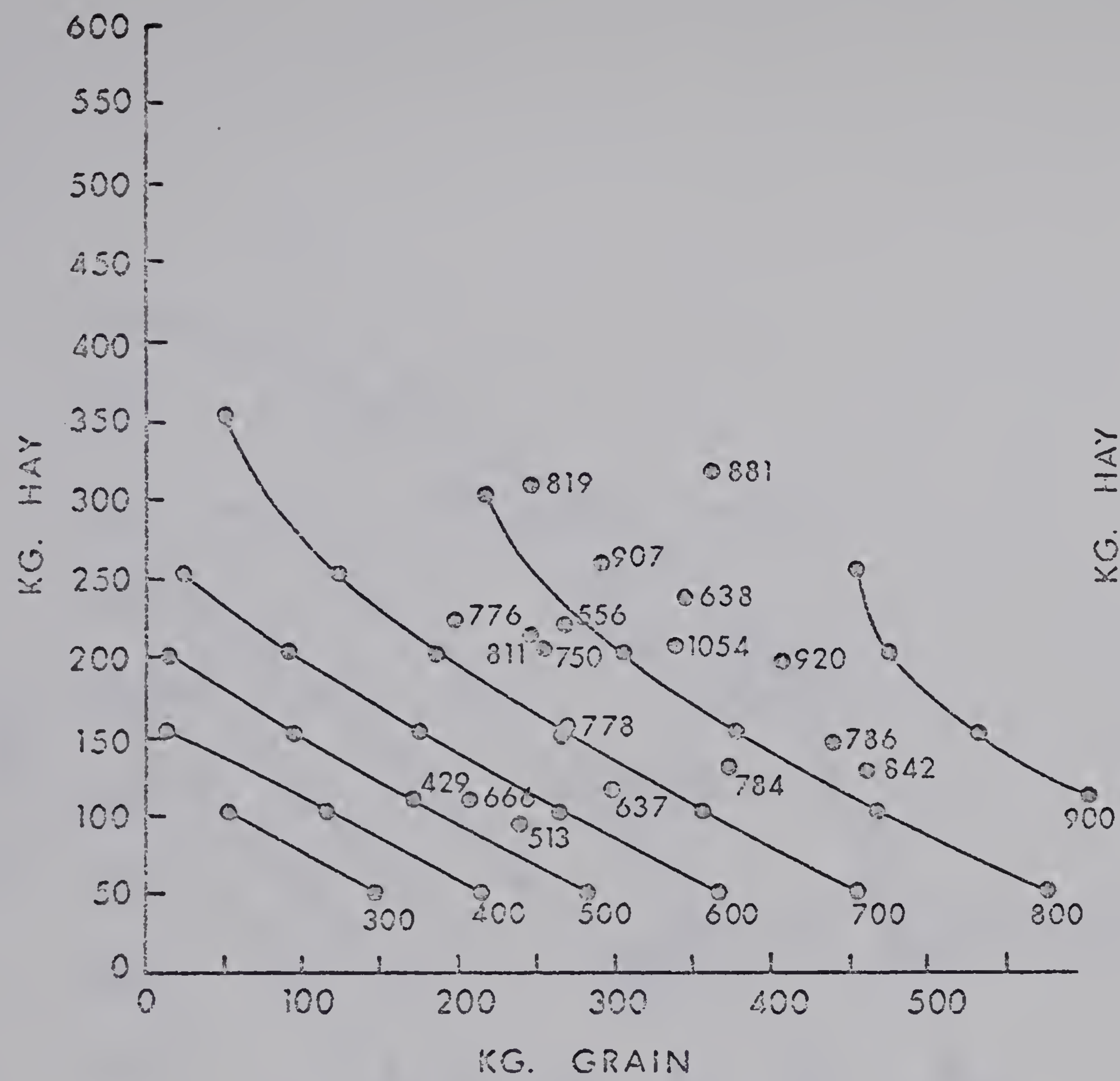

Figure 12

ISOQUANTS FOR EQUATION 22, PERIOD TWO TO FIVE

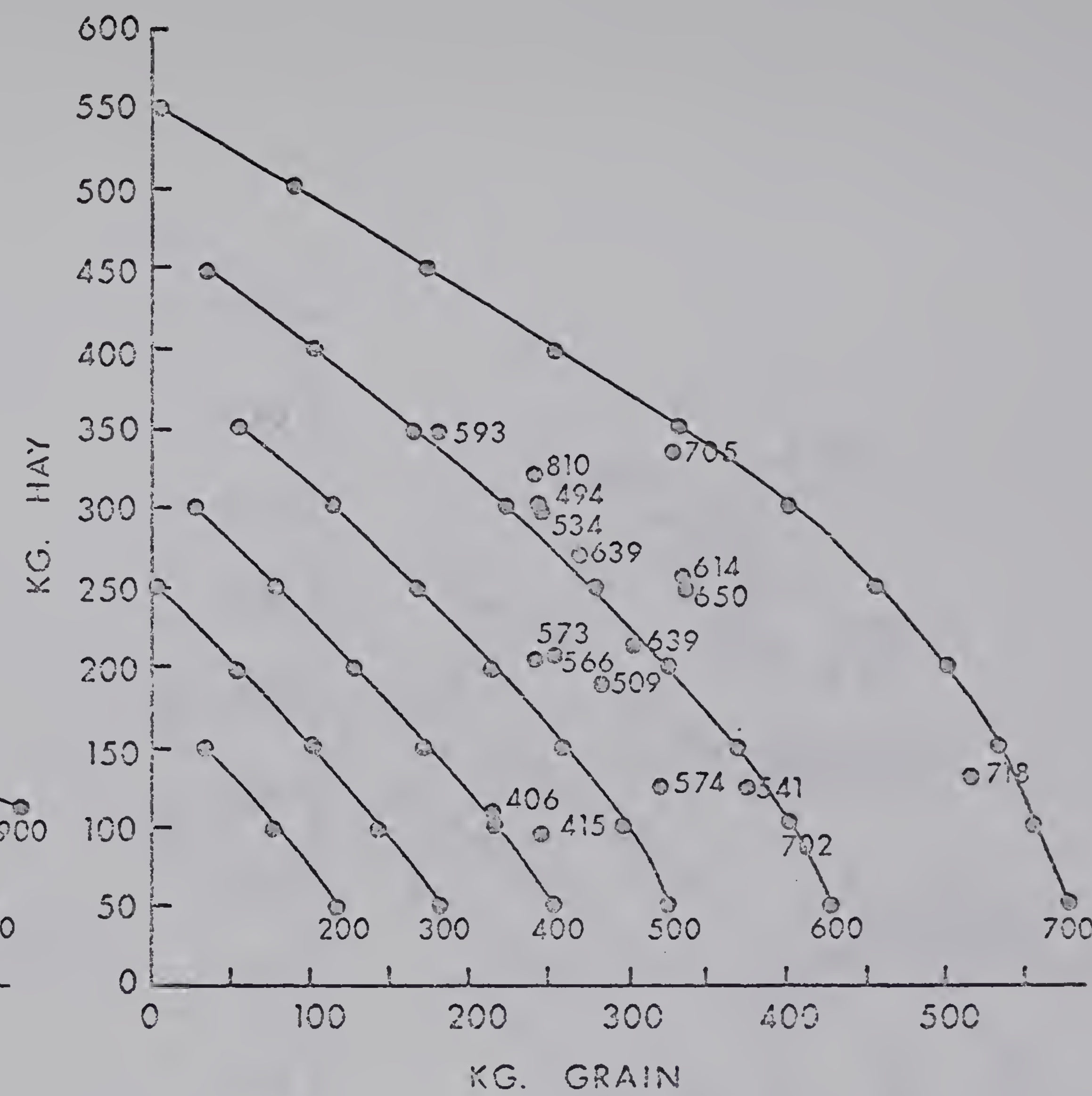

Figure 13

ISOQUANTS FOR EQUATION 25, PERIOD 14 TO 17

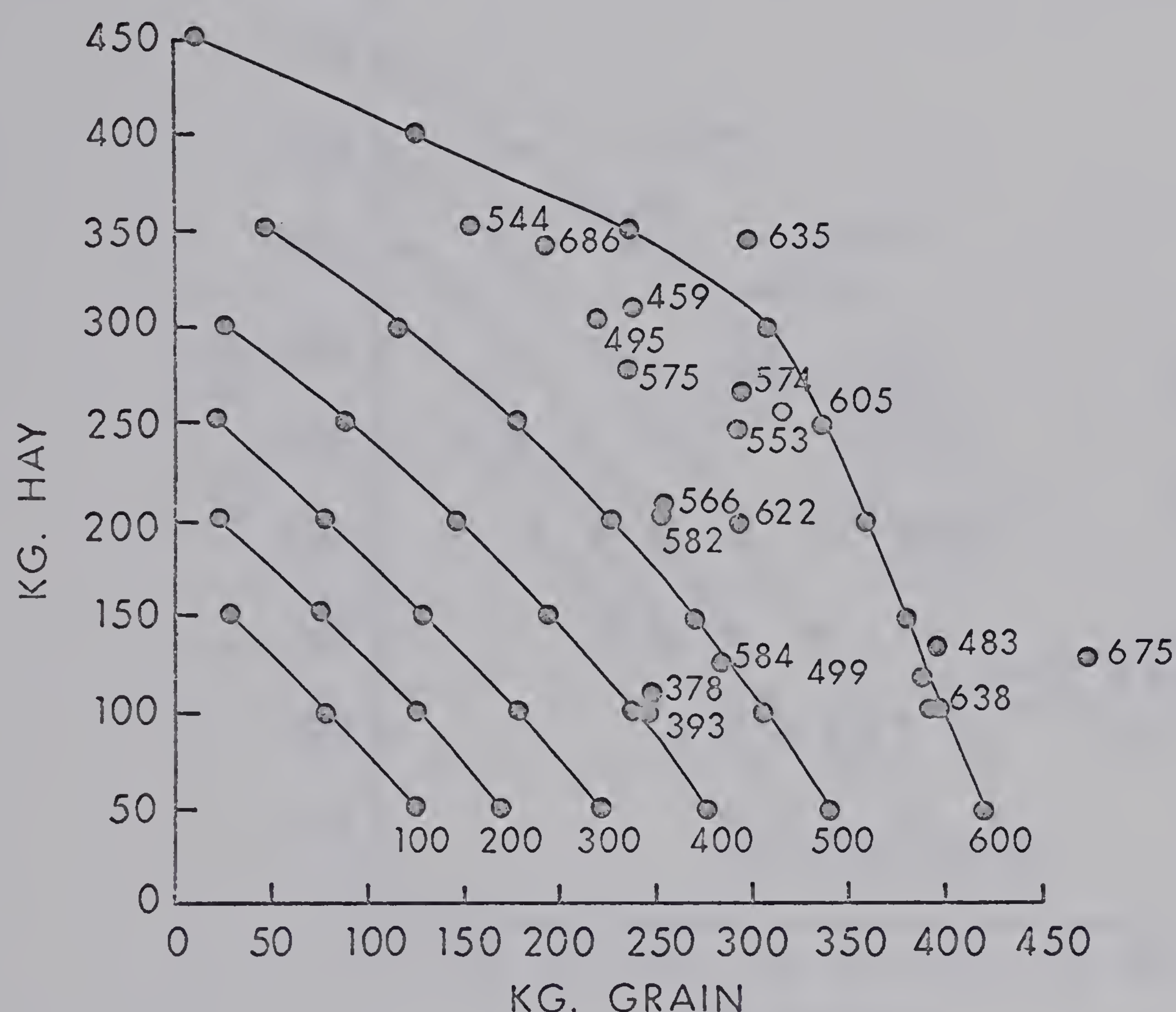

Figure 14

ISOQUANTS FOR EQUATION 26, PERIOD 18 TO 21

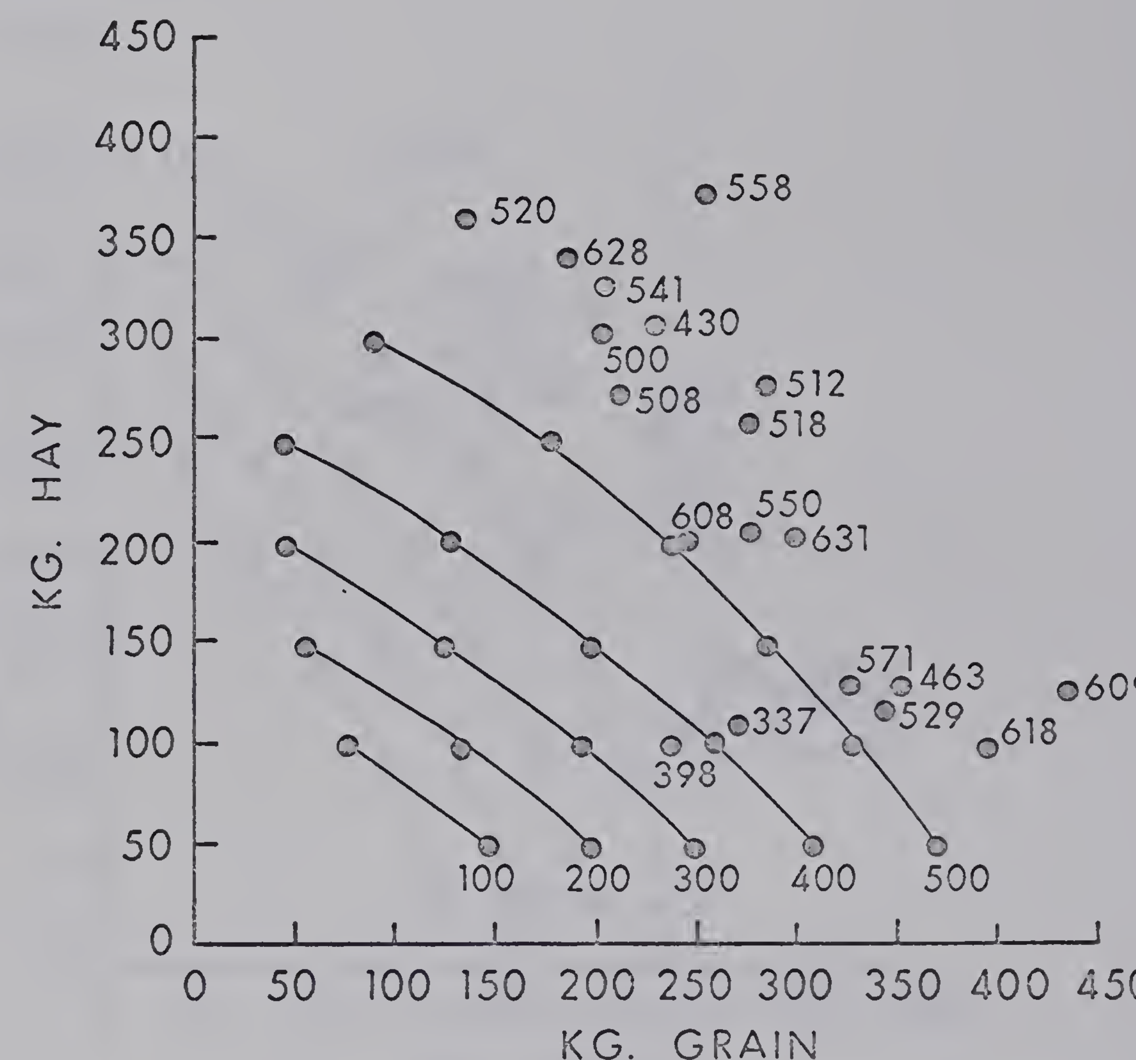

Figure 15

ISOQUANTS FOR EQUATION 27, PERIOD 22 TO 25

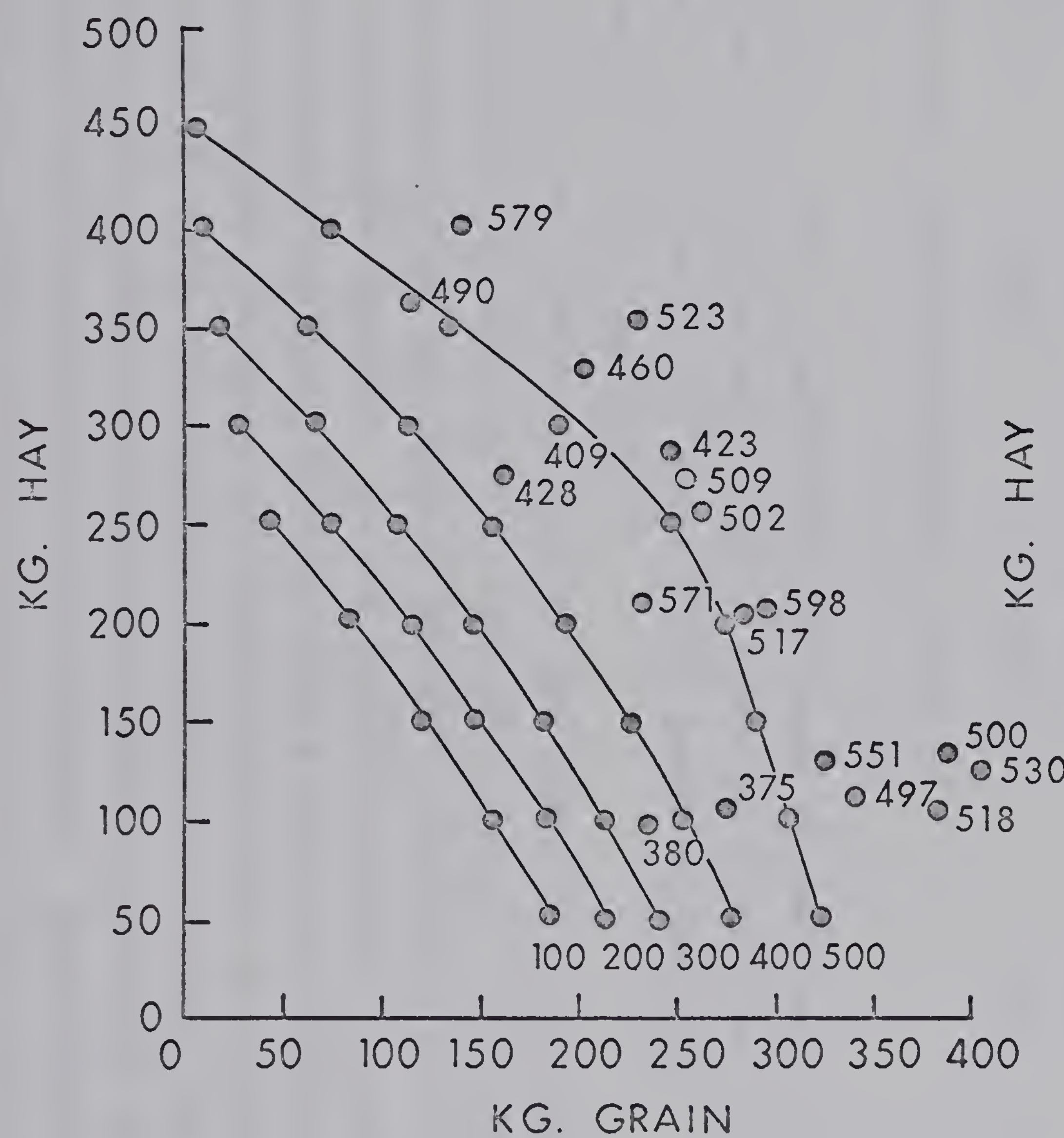

Figure 16

ISOQUANTS FOR EQUATION 28, PERIOD 26 TO 29

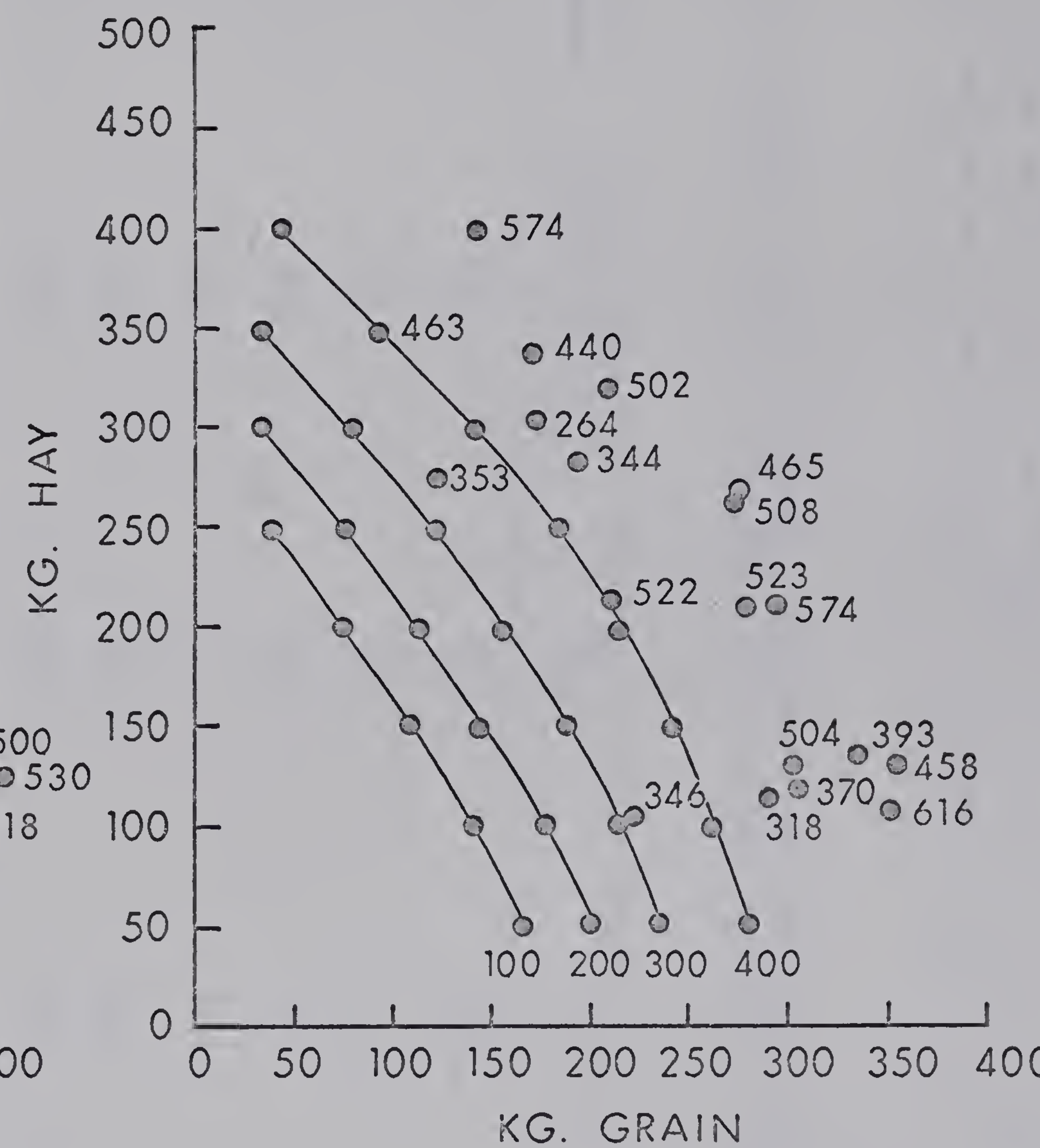

Figure 17

ISOQUANTS FOR EQUATION 30, PERIOD 34 TO 37

of the observations on the other five (Figures 13 to 17) did not tend to be as evenly spaced as those in Figure 12. In these figures the

Table 17

MAXIMUM PHYSICAL PRODUCT AND THE REQUIRED LEVELS OF HAY AND GRAIN FOR THE MAXIMIZING FUNCTIONS WITH FAT CORRECTED MILK AS THE DEPENDENT VARIABLE

Period In Weeks	Equation Number	Maximum physical production Kg	Quantity of hay kg	Quantity of grain kg
2 to 5	22	956	128	807*
14 to 17	25	725	250	601*
18 to 21	26	605	255	315
22 to 25	27	541	326	204
26 to 29	28	504	272	254
34 to 37	30	465	268	275

* These rations were beyond the capacity of a cow

observations tended to form a band which was linear or even concave to the origin across the isoquant map. Higher levels of production did not tend to be concentrated in the areas of the bands representing larger total intakes of hay and grain.

The maximum physical product as determined from the marginal productivity equations decreases in each successive period. The quantities of hay and grain were within the range of possible consumption by a cow for the last four periods analysed. In the first two periods the quantities determined were greater than the quantities that a cow would normally have been expected to consume. In these two periods even though the quantities of feed were above the limits of expected consumption by the cow, the two quantities of milk produced were below the amounts produced by some of the cows during the given periods on the experiment. While producing over the determined quantities of milk on the experiment, the cows consumed lower quantities of hay and grain.

Production Functions With Solids-Corrected Milk Dependent

The production functions for the nine grouped periods were determined with solids-corrected milk as the dependent variable (Table 18). Of the nine equations, five produced maximizing functions, while the remaining four equations had at least one of the signs on their coefficients reversed, and a maximum level of production could not be obtained. The equations that allowed maximum levels of production were from the periods including weeks 2 to 5, 18 to 21, 22 to 25, 26 to 29, and 34 to 37. The squares of the multiple correlation coefficient varied from the corresponding period equation with fat-corrected milk dependent. The change in the amount of variation explained by the solids-corrected milk compared to fat-corrected milk was not consistent sometimes being higher for solids-corrected milk and sometimes lower. The "t" values gave a lower level of significance to most of the coefficients when solids-corrected milk was used as the dependent variable than when fat-corrected milk was used.

Using the five maximizing functions, the marginal analysis produced isoquants similar to the isoquants for the corresponding period when fat-corrected milk was variable. The equation for the period 2 to 5 weeks was the only one producing isoquants that were convex to the origin while the other four equations gave isoquants which were concave to the origin. The marginal rates of transformation of both hay and grain with solids-corrected milk dependent were similar to the marginal rates of transformation obtained when fat-corrected milk was the dependent variable. In some of the cases near the end of an isoquant the magnitudes of the marginal rates of substitution varied but near the center and the relevant range in which the sample is listed the values of the marginal rates of substitution were comparable.

The level of the maximum physical product was only altered slightly (Table 19) as compared to the maximum obtained when fat-corrected milk was the dependent variable (Table 17). The maximum levels reached by the equations with solids-corrected milk dependent were larger than the maximum level of production for the corresponding period when fat-corrected milk was the dependent variable. The rations were very similar between corresponding periods for all but the period

Table 18

REGRESSION EQUATIONS FOR THE NINE GROUPS WITH SCM DEPENDENT, BASED ON THE 18 COWS TO FINISH 37 WEEKS

Period	Equations	R^2	F Value
2 - 5	$SCM = -\ 78.21 + 3.7078H + 1.9381G - .0058H^2 - .0014G^2 - .0010HG$	.546	2.89
	(1.246)* (.496) (.789) (.261) (.131)		
6 - 9	$SCM = 330.93 + 2.6959H - .7938G - .0081H^2 + .0018G^2 + .0042HG$	.655	4.56
	(.982) (.277) (1.066) (.405) (.832)		
10 - 13	$SCM = -\ 147.17 - .5594H + 3.6823G + .0049H^2 - .0035G^2 - .0017HG$	.593	3.49
	(.187) (1.313) (.848) (.953) (.405)		
14 - 17	$SCM = -\ 239.91 + 1.1285H + 3.4247G + .0012H^2 - .0030G^2 - .0032HG$	.503	2.43
	(.452) (1.265) (.277) (.931) (.684)		
18 - 21	$SCM = -\ 675.28 + 3.2712H + 5.1882G - .0012H^2 - .0048G^2 - .0080HG$	.441	1.89
	(1.332) (1.351) (.343) (1.052) (1.350)		
22 - 25	$SCM = -\ 586.07 + 4.0608H - 4.0692G - .0034H^2 - .0031G^2 - .0078HG$	.374	1.43
	(1.383) (.792) (1.023) (.484) (.999)		
26 - 29	$SCM = -\ 1116.63 + 4.9485H + 7.4694G - .0032H^2 - .0078G^2 - .0127HG$	.424	1.77
	(1.461) (1.474) (.921) (1.220) (1.468)		
30 - 33	$SCM = 582.66 - 1.4416H - 2.0770G + .0028H^2 + .0049G^2 - .0013HG$	.393	1.55
	(.276) (.281) (.506) (.478) (.368)		
34 - 37	$SCM = -\ 720.19 + 2.9961H + 5.5207G - .0011H^2 - .0057G^2 - .0082HG$	.768	7.95
	(.995) (1.470) (.309) (1.060) (1.061)		

* These numbers represent the "t" value for the coefficient directly above them

of weeks 2 to 5 in which the solids-corrected milk equation indicated a more reasonable ration to produce the given quantity of milk; however, it would still be beyond the physical capacity of all but the most energetic feeders. Both sets of equations, the fat-corrected milk and solids-corrected milk equations over-fed the cows during the initial periods of the lactation but then approached the actual consumption patterns of the experimental cows for the periods in the latter part of the lactation.

Table 19

MAXIMUM PHYSICAL PRODUCT AND THE REQUIRED LEVELS OF HAY AND GRAIN FOR THE MAXIMIZING FUNCTIONS WITH SOLIDS CORRECTED MILK AS THE DEPENDENT VARIABLE

Period Weeks inclusive	Maximum Physical Production	Quantity Of Hay	Quantity Of Grain
2 to 5	966	270	576
18 to 21	597	248	335
22 to 25	563	351	215
26 to 29	509	284	247
34 to 37	488	261	295

Factors Affecting the Significance of Regression

Partial regression consists of determining coefficients for the regression equation such that the square of the deviations of the observations from the regression values are minimized.[1] The form of the selected regression equation affects the sum of squares of the deviation. For this reason the quadratic regression equation was selected as most representative of the expected behaviour of the observations as outlined in Chapter 2. Even though the quadratic

[1] R.L. Anderson and T.A. Bancroft, Statistical Theory in Research New York: McGraw-Hill Book Company, Inc., pp. 191 - 206.

regression equation was selected the significance with which it predicted the response was low as indicated by the F values (Table 10). Two of the factors in the experiment that affect the significance of the regression equation include cow numbers and experiment size relative to the possible size. The number of cows was probably the most restrictive of the two factors. The three factor factorial design results in 27 experimental units. Twenty seven cows are required to place one cow in each experimental unit and to put a higher number of cows in each experimental unit would require a like multiple of 27 to give the total number of cows required. A larger number of cows could also have been used to give a wider range of hay and grain consumption upon which the regression and marginal analysis would be based. The broader base could also be obtained by altering the existing experiment to give fewer cows which were actually on full feed. During the early weeks of the experiment the cows assigned to the 120 and 100 percent energy rations were essentially on the same ration, as the cow's stomach capacity prevented complete consumption of the alloted hay and grain. The common level of consumption prevented a wide base on which the coefficients could be determined. A reduction of the energy level to a lower level, say 80 percent,would have accentuated the problem of the reduction in cow numbers as the lactation length increased.

The length of this study was another factor which affected the results. As the level of production of the cows fell they were removed from the experiment. Maintaining the cows on a low energy ration for the entire lactation resulted in their developing a more rapid decline in rate of production than the other energy levels. The rate of decline over time during the later period of the study may have become the dominant factor affecting the level of production rather than the level of hay and grain consumption. During the later periods of the lactation the cows were being fed at a nutritional level such that most were gaining weight. The weight gains indicate that the time influence,not the nutritional level,was the more important factor in determining the production level.

CHAPTER VI

PRICE ANALYSIS AND ECONOMIC OPTIMAL RATIONS

During the previous analysis the only reference to a level of production was to the calculated maximum physical product for each of the periods. An analysis of this form gives rise to the question is maximum production necessarily the economic optimum level of production? Is the return from the last unit of milk produced sufficient to cover the cost of producing it? The incorporation of the price of hay and grain as the resources and the price of milk as the product into the marginal analysis will indicate the positions of optimum economic production.

Determination of the Prices

Price of Milk

The price of milk used as the average price was eleven cents per kilogram. Price corresponded to \$4.90 per hundredweight and approximated an average blend received by fluid milk producers or a subsidized price for the shipper of manufacturing milk. The price range selected was two cents per kilogram above or below the average price. The lower level price, nine cents per kilogram or \$4.08 per hundredweight, approximated a price of unsubsidized manufacturing milk or third class milk for the fluid milk producer. The top of the range, thirteen cents per kilogram or \$5.90 per hundredweight, approximated the price of quota milk in the Alberta fluid milk market.

Price of Hay

The prices used for hay were 2, 3, and 4 cents per kilogram. They correspond to \$18, \$27, and \$36 per ton. Two cents per kilogram represented the cost of homegrown roughage overlooking any opportunity cost which may have been present. Three cents per kilogram represented an average purchase price of good quality alfalfa or alfalfa-bromegrass hay. The upper limit of 4 cents per kilogram represented a price which could have been paid for hay during a period of hay shortage in any season or when buying stored hay in the late winter.

Price of Grain

The determination of a range for the price of grain was more difficult. The grain used in this experiment represents a complete concentrate feed. The inclusion of the prices of all concentrate components when considering their prices on a yearly basis seemed to cancel any variation in the price of the complete concentrate. The price determined in this manner was approximately 6 cents per kilogram or $54 per ton. Six cents per kilogram was a reasonable price to use as the average. In developing the range of prices, the grain components of the concentrate were calculated at the production cost to the farmer. This computation gave a value of 5 cents per kilogram or $45 per ton. This price situation would have approximated a farmer selling his own grain through the milk his cows produced. The upper range of the price of grain was set at 7 cents per kilogram or $64 per ton. This price may have only been realistic if all the component prices peaked at the time the ration was being prepared.

Theoretically the determined prices outlined in Table 20 could have been combined in any possible combination; however, some combinations would not be realistic. Only under extreme fluctuations in the prices could the price of one of the components of the analysis be at the opposite end of the range from the other components.

Table 20

PRICES OF HAY, GRAIN, AND MILK AS USED IN THE PRICE ANALYSIS GIVEN IN CENTS PER KILOGRAM AND ALSO THE USUAL METHOD OF MEASUREMENT IN ALBERTA

Price Level	Price of Milk ¢/kg.	Price of Hay ¢/kg.	Price of Grain ¢/kg.	Price of Milk $/cwt.	Price of Hay $/ton	Price of Grain $/ton
Lower	9	2	5	4.08	18	45
Average	11	3	6	4.99	27	54
Higher	13	4	7	5.90	36	64

Economic Optimum Combinations and Level of Production

The economic optimal position of production at seven different price combinations was determined for each of the four-week periods. The price combinations represented a change in the price of hay between the first and the second combinations, a change in the price of grain between the third and the fourth combinations, a change in the price of milk between the fifth and sixth combinations, and the seventh combination of prices represented the average price for each of the components. The expected response from a change in price was for the use of a resource to be reduced as the price of the resource increased and for the quantity of milk produced to increase as the price of milk increased or vice versa for both cases. The economically optimal rations only responded as indicated for the four-week period two to five weeks (Table 21). In the five other periods the changes in the optimal rations varied (Table 29). The optimal rations were altered differently than expected due to the decreasing pattern of the marginal rates of substitution of the last five of the four-week groups. In all the groups except for period, weeks 22 to 25, the change in the economic optimal position was to increase production as the price of milk increased.

Table 21

CHANGES IN THE ECONOMIC OPTIMAL LEVEL OF PRODUCTION OF FAT-CORRECTED MILK AND THE QUANTITIES OF HAY AND GRAIN REQUIRED TO PRODUCE THE MILK DURING THE TWO-TO FIVE-WEEK PERIOD

Price Combination	Price of Hay	Price of Grain	Price of Milk	Kg. Milk	Kg. Hay	Kg. Grain
1	2	6	11	819.4	281.9	259.6
2	4	6	11	844.5	246.4	323.5
3	3	5	11	875.5	232.2	394.5
4	3	7	11	780.6	296.1	188.5
5	3	6	9	771.8	294.9	175.5
6	3	6	13	867.8	242.8	371.8
7	3	6	11	832.8	264.1	291.5

The change in the economic optimal ration as a function of time was shown by comparing the results of price combination seven for each of the four-week periods(Table 22). The level of fat-corrected milk produced decreased with each later period in lactation. Omitting the four-week period, weeks 2 to 5, there is a gradual increase in the amount of hay consumed and a decline in the quantity of grain consumed.

Table 22

THE ECONOMIC OPTIMAL LEVEL OF FAT-CORRECTED MILK OUTPUT AND THE CORRESPONDING RATIONS FOR PRICE SITUATION 7* FOR THE INDICATED FOUR-WEEK PERIODS.

Four Week Period	FCM kg	Hay kg	Grain kg
2 to 5	832.8	264.1	291.5
14 to 17	683.5	98.6	525.0
18 to 21	595.1	195.9	349.0
22 to 25	552.2	219.1	297.5
26 to 29	498.7	227.6	257.2
34 to 37	455.4	228.3	260.0

* Price of milk = 11¢/kg., price of hay = 3¢/kg., and price of grain = 6¢/kg.

A price analysis was also conducted using the five maximizing equations obtained with solids-corrected milk as the dependent variable. Using the same price combinations and the same system of analysis the results for the four week periods behaved similarly to the corresponding periods with fat-corrected milk dependent (Table 30). In this analysis the levels of production did not decline from period to period as consistently as was the case with fat-corrected milk dependent. The four-week period, weeks 34 to 37, actually predicted higher levels of production than the period weeks 26 to 29.

Comparison of the Return Above Feed Costs

With calculations of the economically optimal rations completed the two questions posed in the introduction can be answered. Using price combination 7 of Table 21 the return above feed costs have been calculated for the maximum level of production and the economically optimum ration for the period including weeks two to five. The maximum production level gave a return above feed cost of $52.90 compared to a return of $66.20 above feed costs for the economically optimum ration. These calculations show a difference of $13.30 over the four-week period, weeks two to five, thus maximum production from the cow was not economical production. The second question was about the recommended rations and their economic potential. Two rations were prepared based on the National Research Council [1] recommendations. One ration consisted of .75 kilograms of hay per 100 kilograms of body weight, and the second consisted of 1.5 kilograms of hay per 100 kilograms of body weight with both then supplemented to the recommended energy level with a grain mixture. In calculating these rations, it was assumed that the production level was the economically optimal level, 832.8 kilograms, and that the body weight of the cows was the average weight of the cows on test for the period, 620 kilograms. The rations thus calculated were 130.2 kilograms of hay with 510.2 kilograms of grain for the .75 kilograms of hay ration and 260.4 kilograms of hay with 421.4 kilograms of grain for the 1.5 kilograms of hay ration. The return above feed costs for these rations was $58.39 and $61.12 for the .75 kilograms hay and the 1.5 kilograms of hay rations, respectively. These returns compare to $66.20 from the optimal economic ration and $52.90 for the maximum production ration.

Another question that can now also be answered is should the ration be changed as the price of the feed components or the price of milk change? The return above feed cost was calculated for each of the economic rations

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1 National Research Council.
Nutrient Requirements of Dairy Cattle p p.2 - 6

for the seven price combinations from Table 21 and is compared to the two rations based on .75 kilogram of hay and 1.5 kilogram of hay in Table 23. The returns for the optimum economic ration were calculated based on the production feed levels specified. The returns of the two computed rations were calculated based on the amounts of hay and grain that the two rations determined necessary for the production of 832.8 kilograms of milk, the economic optimum level consistent with price situation seven.

The last two columns of Table 23 represent the increased return over the feed cost obtained from the use of the economic optimum ration rather than either of the two computed rations. The calculations indicate that these was a dollar advantage of from $6.89 to $11.88 for changing the ration being fed as the price of the components changed.

Table 23

COMPARISON OF THE RETURN ABOVE FEED COSTS FOR THE DIFFERENT PRICE COMBINATION AND RATIONS

Price	Return above feed costs of			Economic optimum minus	
Combination	.75 Kg hay ration	1.5 Kg hay ration	Economic optimum ration	.75 Kg. hay ration	1.5 Kg. hay ration
1	58.39	61.12	68.92	10.53	7.79
2	55.79	55.91	63.63	7.84	7.72
3	62.19	62.73	69.62	7.43	6.89
4	51.99	54.30	63.79	11.80	9.49
5	40.43	41.85	50.08	9.65	8.23
6	73.74	75.16	83.22	9.48	8.06
7	57.09	58.51	66.20	9.11	7.69

CHAPTER VII

CONCLUSION

The Objectives and Procedure

The study was undertaken as part of a larger program co-ordinated by the Organization for Economic Co-operation and Development. The objectives were to determine the economic relationship that could be applied to the dairy industry under similar conditions as those found in Alberta. It was also desired to determine the relationship of the existing standards for feeding dairy cattle to the optimum economic relationships determined from this study.

Twenty-seven Holstein cows were divided into three groups and fed roughage .75, 1.50 or 2.5 kilograms per 100 kilograms of body weight. Each group was then subdivided into three lots and fed grain to supplement the hay ration up to 90, 100, or 120 percent of National Research Council recommendations based on the previous weeks production. The data collected were the kilograms of hay and grain consumed and milk produced for each day. These amounts were then totaled over four week periods, and the milk production standardized to a four percent fat-corrected base and also to a solids-corrected base.

The production function analysis was carried out using a quadratic function with corrected milk production as the dependent variable and the quantities of hay and grain as the independent variables. From these, production functions were computed for nine periods of four weeks covering the weeks from week 2 to week 37. Marginal analysis was then to be carried out on each of the production functions. The marginal analysis was then used to determine the optimum economic rations using prices representative of those which are found in Alberta. This analysis was further extended to compare the return over feed costs of the optimum economic ration and two rations based on the National Research Council requirements. This comparison was to determine if the feeding standards as outlined by the National Research Council approached the economic ration.

The Results

Uniformity of lactation length was the major problem that arose in connection with the data. The first cow was removed from the test after 20 weeks, and only 11 of the 27 cows completed 44 weekly records. Lactation length of the cows was related to each of the variables in the experimental design. On the basis of the three production groups into which the cows were initially divided, the group designated as heifers had the longest average lactation length, followed by the high producing group, and then the designated low producers with the shortest average lactation length. When the lactation lengths were compared among the energy groups, the 120 and 100 percent groups showed little difference, but the 90 percent groups had the shorter lactation by seven weeks. Lactation length was also affected by the amount of hay in the ration, with the higher levels of hay being associated with the shorter lactations.

Only the energy groups provided significant differences in average weekly decline in milk production. The 90 percent energy group declined over time more rapidly than did either the 100 or 120 percent energy groups. The 100 percent and 120 percent groups declined at almost identical rates so that their differences in production were mostly affected by the maximum level of weekly production reached near the sixth week of the lactation. Using fat-corrected, solids-corrected, or un-corrected milk as the dependent variable, the 100 percent energy group attained the highest weekly average. When solids-corrected milk was the dependent variable, the 120 and 90 percent groups attained similar maximum levels. However, when fat-corrected or un-corrected milk was the dependent variable, the 120 percent energy group fell below the maximum weekly level of the 90 percent energy group. By the end of the lactation the 100 percent group had the highest average weekly production level, followed by the 120 percent group, then the 90 percent energy group with the lowest average lactation length. A factor that may have contributed to the faster decline in production was the body weight change of the 90 percent energy group as compared

to the body weight changes of the other two groups. The 90 percent energy group lost 48 kilograms, while the 100 percent energy group lost 29 kilograms and the 120 percent energy group gained 38 kilograms.

The initial regression analysis with the weekly data gave only 18 of 44 equations of a maximizing form, and this analysis was not continued. Next, the data were grouped into four-week periods commencing at week two and continuing to week 37. This analysis only produced maximizing equations for six of the periods. The significance of the coefficients determined was low with only 17 of a possible 30 reaching the 80 percent significance level. A further analysis was conducted similar to the grouped analysis including a variable indicating weight change during the period. This analysis only gave one weight change coefficient significant to the 80 percent level and did not greatly increase the explanatory power of the equations.

Problems were also encountered in the marginal analysis section. Only in the two to five week period did the regression equation produce isoquants convex to the origin indicating increasing marginal rates of substitution of the hay and grain. In the other periods the isoquants were concave to the origin indicating decreasing marginal rates of substitution. Economic optimum rations were defined for all periods, but only in the two-to five-week period did the rations change as was expected. The inconsistency of the ration changes resulted from the isoquants being concave to the origin rather than convex as in production theory. The optimal economic ration was calculated for each of the six periods that had maximizing regression equations with fat-corrected milk dependent. Medium prices were used for the feeds and for milk. The rations indicated a gradual increase in hay consumption and a decrease in grain consumption over time. This price analysis was also conducted with solids-corrected milk as the dependent variable, but the results were more inconsistent than with fat-corrected milk dependent. The optimum rations changed with no pattern, and the optimum levels of milk production did not consistently decline with time as was expected.

The calculation of the return above feed costs showed that feeding on the basis of the economic optimum ration produces the highest return.

Using the feeding standards as a nutritional basis provides more economical rations than feeding for maximum production. The return above feed costs also showed the importance of adjusting the ration as the price of either the feed components or of milk change. The advantage of altering the ration with price changes was more favourable as the price of the feed components increased. The results have answered the questions concerning the economic potential of the present feeding standard. The results showed that these standards do provide a more economical ration than feeding for maximum production but still do not provide as economical a ration as one calculated on the basis of the relative prices of the feed components and their marginal productivities.

Implications of the Study

The study has shown the need for the development of a feeding system that incorporates factors to allow for substitution between the feed components as their relative prices change. To make recommendations from this study would require the cows to which the rations would apply to possess the average characteristics of the cows in this study. Further study would have to be carried out to provide a set of data for cows of different productive capacity levels and for different periods of the lactation. These different sets of data are required to compensate for the different marginal productivities of the feed components as determined by the cows inherent capacity to produce or by the time position during the lactation.

When expanding the study, some alteration in design should be incorporated to alleviate the long period on a low energy ration. The lactation could be divided into 10 periods of 30 days approximating monthly intervals. Each of the 30-day periods could then be divided into a 16-day period and two periods of 7 days each. During the first 16-day period the cows could all be fed on a constant high energy ration and the productive capacity of the cows determined. The cows then could be allotted to that section of the experiment designed for the cows of that productive level and further assigned, at random, to receive a specific ration made up of some predetermined level of hay and grain. The next 7-day period would be to allow for the cow to adjust to the

ration, followed by a 7-day test period during which the concumption and production data are obtained. Following this test period the cows would return to the constant ration at the beginning of the next 30-day test period. During the next period the cow would be assigned to the same productive group and to the same ration. In this way the cows allotted to low energy rations would only be on the experimental rations for 14 of the 30 days in the period, and the rate of decline of these cows may be slowed and longer lactations may be obtained. Any suggested changes in the experiment must be evaluated on the basis of the additional information provided in relation to the added cost of the larger or more complex design.

APPENDIX

Table 24

COW BODYWEIGHTS IN KILOGRAMS AT THE DESIGNATED WEEK

Wk.	Cow # 809	343	410	347	233	451	503	345	504	127	506	501	124	232	507
1	626	620	568	647	670	536	462	661	499	857	546	506	798	662	474
5	595	635	520	650	670	541	456	620	455	798	537	495	747	584	451
9	590	578	514	639	662	550	471	639	453	790	484	503	765	597	455
13	590	599	527	653	672	565	500	645	475	765	510	503	730	605	460
17	604	645	525	655	665	576	485	605	470	728	511	492	737	609	475
21	622	634	533	654	664	578	485	613	483	698	514	492	736	564	481
25	617	646	524	654	662	591	523	630	505	645	531	492	744	571	498
29	607	658	558	648	653	610	523	620	505	659	531	510	725	571	498
33	640	658	558	648	653	625	528	630	505	657	531	540	740	565	525
37	615	658	558	648	653	643	540	646	535	663	550	555	740	565	525
41	637	735	570	670	660	663	540	646	560	663	590	563	775	575	574

Wk.	Cow # 913	237	403	235	344	130	810	707	342	348	022	408
1	737	711	579	661	663	647	757	735	574	700	747	508
5	690	664	560	635	624	585	674	667	572	699	668	475
9	695	660	556	634	594	595	682	681	535	697	652	475
13	678	684	541	622	595	607	699	649	520	690	625	479
17	664	671	545	575	588	619	689	666	530	692	595	511
21	671	695	561	583	562	600	665	675	535	692	602	
25	671	700	569	558	554	592	675	661	520	679	597	
29	655	713	590	557	554	585	680	655	533			
33	655	721	590	563	554	573	676					
37	662	750	590									

Table 25

BODYWEIGHT CHANGES IN KILOGRAMS FROM THE BEGINNING TO THE END OF THE PERIOD INDICATED

Cow # Period	809	343	410	347	233	451	503	345	504	127	506	501	124	232	507
2 - 5	-31	+15	-48	3	0	5	- 6	-41	-44	-59	- 9	-11	-51	-78	-23
6 - 9	- 5	-57	- 6	-11	- 8	9	15	19	- 2	- 8	-53	8	18	13	4
10 - 13	0	21	13	14	10	15	29	6	22	-25	26	0	-35	8	5
14 - 17	14	46	- 2	2	- 7	11	-15	-40	- 5	-37	1	-11	7	4	15
18 - 21	18	-11	8	- 1	- 1	2	0	8	13	-30	3	0	- 1	-45	6
22 - 25	- 5	12	- 9	0	- 2	13	38	17	22	-53	17	0	8	7	17
26 - 29	-10	+12	34	- 6	- 9	19	0	-10	0	14	0	18	-19	0	0
30 - 33	33	0	0	0	0	15	5	10	0	- 2	0	30	15	- 6	27
34 - 37	-25	0	0	0	0	18	12	16	30	6	19	15	0	0	0
38 - 41	22	77	12	22	7	20	0	0	25	0	40	8	35	10	49
Net change	11	115	2	23	-10	127	78	-25	61	-194	44	57	-23	-87	100

Cow # Period	913	237	403	235	344	130	810	707	342	348	022	408
2 - 5	-47	-47	-19	-26	-39	-62	-83	-68	- 2	- 1	-79	-33
6 - 9	5	- 4	- 4	- 1	-30	10	8	14	-37	- 2	-16	0
10 - 13	-17	24	-15	-12	1	12	17	-32	-15	- 7	-27	4
14 - 17	-14	-13	4	-47	- 7	12	-10	17	10	2	-30	32
18 - 21	7	24	16	8	-26	-19	-24	9	5	0	7	--
22 - 25	0	5	8	-25	- 8	- 8	10	-14	-15	-13	- 5	--
26 - 29	-16	13	21	- 1	0	- 7	5	- 6	13	--	--	
30 - 33	0	8	0	6	0	-12	- 4	--	--			
34 - 37	7	29	0	--	--	--	--					
Net change	-75	39	11	-98	-109	-74	-81	-80	-41	21	-150	3

Table 26

WEEKLY REGRESSIONS WITH FAT-CORRECTED MILK DEPENDENT, USING ALL COWS HAVING RECORDS DURING THE WEEK

Week	No. of Cows	Value Of The Intercept	Hay Coefficent	Grain Coefficent	Hay Squared Coefficent	Grain Squared Coefficent	Hay-Grain Coefficent	R^2	F
1	27	110.885	.9049 (.379)	-2.2791 (1.188)	.0236 (.787)	.0599 (3.358)	-.0549 (1.316)	.652	7.87
2	27	-441.299	11.1081 (1.969)	10.5057 (2.001)	-.0607 (1.414)	-.0448 (1.896)	-.0751 (1.659)	.342	2.17
3	27	- 83.311	2.4743 (.923)	4.2314 (1.532)	-.0041 (.232)	-.0145 (1.164)	-.0169 (.757)	.478	3.84
4	27	- 10.754	2.1094 (.980)	2.7109 (1.556)	-.0080 (.512)	-.0096 (1.164)	-.0061 (.482)	.484	3.94
5	27	76.530	.4892 (.310)	.6039 (.523)	.0034 (.304)	.0036 (.593)	.0011 (.123)	.672	8.62
6	27	157.276	.6883 (.386)	-1.7304 (1.893)	-.0053 (.398)	.0141 (2.592)	.0121 (1.266)	.590	6.04
7	27	137.149	.5238 (.333)	-1.2966 (.869)	-.0044 (.367)	.0120 (1.455)	.0117 (1.283)	.643	7.56
8	27	191.666	.5882 (.436)	-3.4500 (1.926)	-.0080 (.960)	.0246 (2.329)	.0202 (2.085)	.781	14.99
9	27	125.169	-.4921 (.379)	- .8229 (.550)	.0051 (.697)	.0102 (1.314)	.0113 (1.381)	.760	13.29
10	27	- 22.032	.5473 (.325)	2.5508 (1.199)	.0028 (.311)	-.0082 (.803)	.0003 (.027)	.609	6.55

Table 26 Continued

Week	No. of Cows	Value Of The Intercept	Hay Coefficent	Grain Coefficent	Hay Squared Coefficent	Grain Squared Coefficent	Hay-Grain Coefficent	R^2	F
11	27	57.870	.3752 (.208)*	- .0392 (.018)	.0014 (.142)	.0079 (.762)	.0073 (.519)	.654	7.96
12	27	55.093	.2032 (.126)	.2740 (.157)	.0002 (.023)	.0024 (.324)	.0136 (1.091)	.716	10.57
13	27	23.073	.4578 (.339)	1.0044 (.562)	.0009 (.111)	.00004 (.005)	.0064 (.598)	.707	10.14
14	27	-40.797	1.2326 (.817)	2.1364 (1.205)	-.0012 (.172)	-.0040 (.479)	-.0029 (.252)	.727	11.18
15	27	-27.562	.5139 (.375)	2.4730 (1.774)	.0045 (.614)	-.0073 (1.268)	-.0021 (.194)	.755	12.94
16	27	60.467	-.3724 (.287)	.8530 (.531)	.0054 (.732)	-.0012 (.155)	.0090 (.803)	.696	9.63
17	27	-142.96	2.6804 (1.566)	4.1369 (2.035)	-.0058 (.713)	-.0135 (1.487)	-.0170 (1.176)	.707	10.13
18	27	53.25	.4236 (.278)	.5688 (.312)	-.0020 (.262)	-.0005 (.062)	.0086 (.637)	.649	7.75
19	27	- 87.85	1.4789 (.733)	3.7330 (1.606)	.0006 (.065)	-.0141 (1.343)	-.0130 (.727)	.690	9.34
20	27	-133.94	2.1088 (1.174)	4.6850 (2.397)	-.0012 (.137)	-.0184 (1.988)	-.0200 (1.331)	.734	11.57
21	26	-112.76	2.1575 (1.254)	3.6122 (1.886)	-.0026 (.312)	-.0107 (1.196)	-.0159 (1.078)	.745	11.66

Table 26 Continued

Week	No. of Cows	Value of The Intercept	Hay Coefficent	Grain Coefficent	Hay Squared Coefficent	Grain Squared Coefficent	Hay-Grain Coefficent	R^2	F
22	26	-100.01	2.5664	3.0097	-.0079	-.0081	-.0129	.743	11.57
			(1.480)	(1.635)	(.917)	(.976)	(.882)		
23	26	-160.17	3.7545	4.1453	-.0143	-.0142	-.0211	.592	5.80
			(1.446)	(1.443)	(1.168)	(1.023)	(.954)		
24	26	-230.00	4.6855	5.6795	-.0171	-.0224	-.0332	.596	5.90
			(2.061)	(2.100)	(1.627)	(1.655)	(1.660)		
25	26	-293.58	5.0628	7.7339	-.0152	-.0393	-.0459	.752	12.17
			(3.255)	(4.694)	(1.818)	(3.925)	(3.838)		
26	25	-218.05	4.6739	5.5579	-.0172	-.0212	-.0353	.743	10.97
			(2.534)	(2.902)	(1.794)	(2.191)	(2.250)		
27	25	-154.31	3.2069	4.7410	-.0083	-.0181	-.0280	.743	10.98
			(1.619)	(2.390)	(.853)	(1.765)	(1.665)		
28	25	-102.73	2.1069	3.6984	-.0023	-.0135	-.0200	.156	11.75
			(1.256)	(2.301)	(.271)	(1.793)	(1.372)		
29	24	-144.59	2.6927	4.9990	-.0034	-.0213	-.0312	.627	6.06
			(1.101)	(1.865)	(.284)	(1.519)	(1.340)		
30	24	- 28.31	1.0044	2.1095	.0014	-.0061	-.0078	.599	5.37
			(.620)	(1.136)	(.144)	(.498)	(.519)		
31	23	-107.76	2.4056	3.3923	-.0050	-.0117	-.0173	.647	6.22
			(.927)	(1.126)	(.362)	(.717)	(.687)		
32	22	- 68.19	.4702	1.4645	.0044	-.0027	-.0011	.669	6.46
			(.384)	(1.219)	(.380)	(.274)	(.127)		

Table 26 Continued

Week	No. of Cows	Value of The Intercept	Hay Coefficent	Grain Coefficent	Hay Squared Coefficent	Grain Squared Coefficent	Hay-Grain Coefficent	R^2	F
33	22	.0648	.1588	1.4440	.0071	-.0006	-.0034	.748	9.52
			(.204)	(1.657)	(1.008)	(.083)	(.499)		
34	21	59.78	-1.0004	-.1124	.0134	.0086	.0100	.676	6.26
			(.399)	(.049)	(.843)	(.694)	(.510)		
35	20	1.86	.2973	.6891	.0064	.0088	-.0017	.703	6.62
			(.110)	(.228)	(.480)	(.525)	(.060)		
36	19	-122.87	1.6872	4.9867	.0018	-.0235	-.0257	.743	7.53
			(1.013)	(1.811)	(.206)	(1.306)	(1.258)		
37	18	- 48.48	1.8327	2.7044	-.0042	-.0103	-.0176	.784	8.73
			(1.475)	(2.326)	(.553)	(1.106)	(1.458)		
38	15	36.29	1.0935	-.2239	-.0026	.0126	-.0055	.511	1.88
			(.291)	(.033)	(.173)	(.297)	(.123)		
39	15	62.83	.6464	-1.3463	-.0026	.0191	.0083	.614	2.87
			(.171)	(.180)	(.174)	(.402)	(.169)		
40	15	- 13.03	.8919	1.5358	.0003	-.0013	-.0074	.545	2.16
			(.296)	(.286)	(.025)	(.037)	(.210)		
41	15	216.33	-2.7209	-5.0496	.0161	.0454	.0384	.506	1.85
			(.830)	(.792)	(.955)	(.980)	(.889)		
42	14	137.68	- 1.5722	-3.0754	.0108	.0326	.0241	.637	2.81
			(.614)	(.800)	(.809)	(1.225)	(.825)		
43	11	-892.07	15.0800	16.3630	-.0570	-.0590	-.1289	.900	9.02
			(1.679)	(1.494)	(1.521)	(1.075)	(1.483)		
44	11	14.97	.7912	.2014	-.0013	.0181	.0029	.917	11.11
			(1.042)	(.154)	(.221)	(1.354)	(.216)		

*These figures represent the "t" values of the coefficents

Table 27

REGRESSION EQUATIONS WITH FCM DEPENDENT, USING THE 18 COWS TO COMPLETE 37 WEEKS INCLUDING WEIGHT CHANGE (W) AS AN INDEPENDENT VARIABLE

Period	Equation Number	Equation	R^2
2 - 5	22A	$FCM = -234.34 + \underset{(1.35)^{1}}{4.2389H} + \underset{(.58)}{2.6096G} - \underset{(.72)}{.0055H^2} - \underset{(.29)}{.0018G^2} - \underset{(.47)}{.0035HG} + \underset{(.28)}{.5126W}$	.534
6 - 9	23A	$FCM = 305.08 + \underset{(1.18)}{3.2610H} - \underset{(.39)}{1.0926G} - \underset{(1.05)}{.0080H^2} + \underset{(.59)}{.0026G^2} + \underset{(.52)}{.0026HG} - \underset{(.20)}{.2230W}$	.670
10 - 13	24A	$FCM = 14.82 + \underset{(.31)}{1.0151H} + \underset{(.62)}{1.7491G} + \underset{(.25)}{.0015H^2} - \underset{(.19)}{.0007G^2} - \underset{(.50)}{.0022HG} - \underset{(.34)}{.3519W}$	.586
14 - 17	25A	$FCM = -146.34 + \underset{(.90)}{1.8251H} + \underset{(.58)}{2.3165G} - \underset{(.47)}{.0002H^2} - \underset{(.33)}{.0014G^2} - \underset{(.69)}{.0034HG} - \underset{(.07)}{.6098W}$	.519
18 - 21	26A	$FCM = -328.83 + \underset{(.90)}{2.9253H} + \underset{(.58)}{2.9403G} - \underset{(.47)}{.0020H^2} - \underset{(.33)}{.0019G^2} - \underset{(.69)}{.0054HG} - \underset{(.07)}{.0757W}$	.434
22 - 25	27A	$FCM = -540.05 + \underset{(1.54)}{4.7581H} + \underset{(.61)}{3.2539G} - \underset{(1.27)}{.0044H^2} - \underset{(.23)}{.0015G^2} - \underset{(1.05)}{.0090HG} - \underset{(.71)}{.7849W}$	.434
26 - 29	28A	$FCM = -1161.17 + \underset{(1.58)}{5.0118H} + \underset{(1.56)}{7.3897G} - \underset{(1.00)}{.0032H^2} - \underset{(1.27)}{.0075G^2} - \underset{(1.54)}{.0124HG} - \underset{(1.71)}{1.8360W}$	.544
30 - 33	29A	$FCM = 752.89 - \underset{(.32)}{2.0128H} - \underset{(.34)}{3.1084G} + \underset{(.50)}{.0034H^2} + \underset{(.48)}{.0063G^2} + \underset{(.39)}{.0063HG} + \underset{(.07)}{.1084W}$	.347
34 - 37	30A	$FCM = -791.59 + \underset{(1.09)}{3.4563H} + \underset{(1.46)}{5.7903G} - \underset{(.44)}{.0017H^2} - \underset{(1.04)}{.0060G^2} - \underset{(1.17)}{.0095HG} - \underset{(.13)}{.1013W}$	.750

[1] Numbers in the brackets are the " *t* " values for the coefficient directly above.

Table 28

EQUATIONS FOR THE MARGINAL PRODUCTS MARGINAL RATES OF SUBSTITUTE, AND ISOQUANTS FOR THE GROUPED EQUATIONS PRODUCING MAXIMIZING FUNCTIONS

Equations	Functions
22A	$\frac{\partial M}{\partial H} = 4.3859 - .0116H - .0036G$
	$\frac{\partial M}{\partial G} = 2.0793 - .0020G - .0036H$
	$\frac{\partial H}{\partial G} = 2.0793 - .0020G - .0036H \,/\, 4.3859 - .0116H - .0036G$
	$G = \frac{-(2.0793 - .0036 \times HAY)}{2 \times - .001}[(2.0793 - .0036 \times HAY)^2 - [4 \times - .001 \times (-165.03 + 4.3859 \times HAY - .0058 \times HAY^2) - FCM])^{\frac{1}{2}}$
25A	$\frac{\partial M}{\partial H} = 1.5919 - .0006H - .0024G$
	$\frac{\partial M}{\partial G} = 2.0421 - .0024G - .0024H$
	$\frac{\partial H}{\partial G} = \frac{2.0421 - .0024G - .0024H}{1.5919 - .0006H - .0024G}$
	$G = -(2.0421 - .0024 \times HAY) + [(2.0421 - .0024 \times Hay)^2 - [4x - .0012 \times (-87.67 + 1.59 \times HAY - .0003 \times HAY^2) - FCM])^{\frac{1}{2}}$
26A	$\frac{\partial M}{\partial H} = 3.0453 - .004H - .0057G$
	$\frac{\partial M}{\partial G} = 3.1276 - .0042G - .0057H$
	$\frac{\partial H}{\partial G} = \frac{3.1276 - .0042G - .0057H}{3.0453 - .004H - .0057G}$
	$G = -(3.1276 - .0057 \times HAY) + [(3.1276 - .0057 \times HAY)^2 - [4 x - .0021 \times (-310.73 + 3.0453 \times HAY - .002 \times Hay^2) - FCM])^{\frac{1}{2}}$

Table 28 (Continued)

Equation

27A

$$\frac{\partial M}{\partial H} = 4.2099 - .0086H - .0069G$$

$$\frac{\partial M}{\partial G} = 2.7120 - .0022G - .0069H$$

$$\frac{\partial H}{\partial G} = \frac{2.7120 - .0022G - .0069H}{4.2099 - .0086H - .0069G}$$

$$G = -(2.712 - .0069 \times HAY) + [(2.712 - .0069 \times HAY)^2 - [4 \times -.0011 \times (-423.48 + 4.2099 \times HAY - .0043 \times HAY^2) - FCM]]^{\frac{1}{2}}$$

28A

$$\frac{\partial M}{\partial H} = 5.2349 - .007H - .0131G$$

$$\frac{\partial M}{\partial G} = 7.4359 - .0152G - .0131H$$

$$\frac{\partial H}{\partial G} = \frac{7.4359 - .0152G - .0131H}{5.2349 - .007H - .0131G}$$

$$G = -(7.4359 - .0131 \times HAY) + [(7.4359 - .0131 \times HAY)^2 - [4 \times -.0076 \times (-1154.33 + 5.2349 \times HAY - .0035 HAY^2) - FCM]]^{\frac{1}{2}}$$

30A

$$\frac{\partial M}{\partial H} = 3.4731 - .0032H - .0095G$$

$$\frac{\partial M}{\partial G} = 5.7891 - .0118G - .0095H$$

$$\frac{\partial H}{\partial G} = \frac{5.7891 - .0118G - .0095H}{3.4731 - .0032H - .0095G}$$

$$G = -(5.4359 - .0095 \times Hay) + [(5.4359 - .0095 \times HAY)^2 - [4 \times -.0059 \times (-794.62 + 3.4731 \times HAY - .0016 \times HAY^2) - FCM]]^{\frac{1}{2}}$$

1 Marginal productivity of hay
2 Marginal productivity of grain
3 Marginal rate of substitution of hay for grain
4 Quantity of grain to maintain the quantity of FCM with the given level of hay - the isoquant equation

Table 29

CHANGES IN THE ECONOMIC OPTIMAL LEVEL OF PRODUCTION OF FAT-CORRECTED MILK AND THE QUANTITIES OF HAY AND GRAIN REQUIRED TO PRODUCE THE MILK

Four Week Period	Price Combination	Price Of Hay	Price Of Grain	Price Of Milk	FCM Kg.	Hay Kg.	Grain Kg.
14 - 17	1	2	6	11	699.6	48.1	575.5
	2	4	6	11	672.0	149.1	474.5
	3	3	5	11	691.0	149.1	512.4
	4	3	7	11	677.2	48.1	537.6
	5	3	6	9	663.1	[illegible]64.9	508.2
	6	3	6	13	695.2	121.9	536.7
	7	3	6	11	683.5	98.6	525.0
18 - 21	1	2	6	11	607.6	171.5	382.0
	2	4	6	11	584.8	220.2	315.9
	3	3	5	11	592.5	228.9	325.8
	4	3	7	11	599.8	162.9	372.1
	5	3	6	9	592.1	168.1	357.8
	6	3	6	13	596.9	215.1	342.8
	7	3	6	11	595.1	195.9	349.0
22 - 25	1	2	6	11	562.5	212.2	319.3
	2	4	6	11	542.5	226.1	275.6
	3	3	5	11	544.5	241.0	270.2
	4	3	7	11	562.3	197.3	324.7
	5	3	6	9	557.9	194.6	319.2
	6	3	6	13	548.9	236.1	282.4
	7	3	6	11	552.2	219.1	297.5
26 - 29	1	2	6	11	503.9	206.4	275.5
	2	4	6	11	495.5	248.8	238.9
	3	3	5	11	498.8	245.8	247.4
	4	3	7	11	499.5	209.3	266.9
	5	3	6	9	496.1	217.3	258.0
	6	3	6	13	500.2	234.6	256.6
	7	3	6	11	498.7	227.6	257.2
34 - 37	1	2	6	11	459.8	207.8	276.4
	2	4	6	11	453.0	248.7	243.5
	3	3	5	11	457.2	244.7	254.4
	4	3	7	11	454.2	211.8	265.5
	5	3	6	9	450.7	220.0	256.4
	6	3	6	13	458.1	234.0	262.5
	7	3	6	11	455.4	228.3	260.0

Table 30

CHANGES IN THE ECONOMIC OPTIMAL LEVEL OF PRODUCTION OF SOLIDS-CORRECTED MILK AND THE QUANTITIES OF HAY AND GRAIN REQUIRED TO PRODUCE THE MILK

Four Week Period	Price Combination	Price Of Hay	Price Of Grain	Price Of Milk	SCM Kg.	Hay Kg.	Grain Kg.
2 - 5	1	2	6	11	943.8	269.4	401.2
	2	4	6	11	942.6	253.2	406.9
	3	3	5	11	959.5	258.4	437.6
	4	3	7	11	924.6	264.2	370.6
	5	3	6	9	917.2	259.8	361.3
	6	3	6	13	958.7	262.4	433.6
	7	3	6	11	943.6	261.3	404.1
18 - 21	1	2	6	11	590.1	182.7	331.4
	2	4	6	11	582.4	225.3	295.8
	3	3	5	11	587.5	221.8	308.3
	4	3	7	11	583.6	186.3	318.9
	5	3	6	9	579.5	194.5	308.9
	6	3	6	13	588.6	210.6	316.9
	7	3	6	11	585.3	204.0	313.6
22 - 25	1	2	6	11	581.6	183.9	337.0
	2	4	6	11	556.6	244.3	261.0
	3	3	5	11	561.5	252.0	265.9
	4	3	7	11	576.9	176.1	332.1
	5	3	6	9	566.5	183.6	317.8
	6	3	6	13	566.5	235.2	286.0
	7	3	6	11	567.7	214.1	299.0
26 - 29	1	2	6	11	508.0	220.9	264.0
	2	4	6	11	500.1	267.1	226.0
	3	3	5	11	503.4	262.8	235.8
	4	3	7	11	503.5	225.2	254.7
	5	3	6	9	500.1	234.3	245.3
	6	3	6	13	504.7	250.7	245.2
	7	3	6	11	503.0	244.0	245.2
34 - 37	1	2	6	11	530.0	269.9	270.8
	2	4	6	11	523.1	311.3	237.5
	3	3	5	11	527.9	307.2	249.7
	4	3	7	11	523.7	274.0	258.6
	5	3	6	9	520.0	282.2	249.0
	6	3	6	13	528.9	296.4	257.7
	7	3	6	11	525.6	290.6	254.2

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